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1970

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PLANNING FOR THE OPTIMIZATION OF RESOURCE ALLOCATION IN
STATE AID TO EDUCATION: AN APPLICATION TO IOWA PUBLIC SCHOOLS

by

Norman Howard Starler

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
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Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Ames, Iowa

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CHAPTER I. INTRODUCTION

Because of concern shown by Schultz (63), Weisbrod (84), Sanyal and Fox (62) and others, the study of the economics of education has become established as an important area of concentration within the field of economics.[1] There are five parts to the study to be reported. A brief review of the relevant literature is given in Chapter II. Many of the writings are treated in greater detail within subsequent chapters at the appropriate points. Chapter III discusses the regional (i.e. state, county and school district) distribution effects of state school aid grants to local districts. Chapter IV is concerned with the educational output effects of state grant programs. The more general problem of the specification of the input-output functions of educational systems is treated in Chapter V. The summary and conclusions of the analysis are presented in Chapter VI.

Chapter III begins the analysis with a discussion of the resource redistribution effects of state aid plans because analyses of these effects have been the traditional framework for the consideration of state grants. Herein treated are some of the standard hypotheses as to the economic effects of state aid plans. Additional analysis undertaken in this chapter deals with the impact of the form of finance on the redistribution effects of state aid plans. Chapter IV develops the output effects of state aid programs and continues work the author began earlier.[2] Because economists

[1]See 5, p. 177, subject designation 21.8 for an exhaustive list of current studies in the economics of education.

[2]This work was on the measurement of educational output. It was reported in 67.

and other researchers have had difficulty in specifying educational production functions, Chapter V takes up the problem of seeking to specify relationships between educational output and input.

McCamley (52) noted that:

> Most of the work that has been done in this area [the economics of education] must be regarded as contributing to the macro-economic aspects of education since it has concentrated largely on the relationship of education to the formation of human capital, on the returns to educational investments, or on the closely related matter of national educational decision making (52, p. 1).

Hence McCamley dealt with micro-economic models of interest to decision-makers at individual universities. Perhaps as a reaction to pressing problems, interest and research in the micro-economic area has heightened. In November, 1967 a symposium on the operations analysis of educational decisions was held at which Keller(39), Judy (36), Koenig (45) and Bowman (12) presented models for decision makers in colleges and universities.

The first micro-economic analysis of the cost and output of primary and secondary education was performed on New York State schools by Herbert Kiesling (43; 44). Other analyses related to expenditures in public primary and secondary education have been reported by Ross (61) and were primarily interested in the relation between expenditures and measures of the educational process as distinguished from the educational product.

Cohn (14) performed a similar analysis for school districts of Iowa. By early 1969, the state of the micro-economic analysis of primary and secondary school systems had advanced to the point where Sisson was offering school districts a programmed analysis in order that they might evaluate their allocation of resources (65).

However, by late 1969, few references to the empirical analysis of the redistribution effects of state aid programs existed. Therefore, as might be expected, studies of the output effects of these programs are not available. The satisfactory specification of educational production functions remains unfinished.

In Chapter II, the literature review, only articles having a direct bearing to the study at hand are discussed. That is, only studies where similar methods of analysis were employed or where similar subject matter was analyzed are cited. For this reason, many names familiar to those working in the economics of education are omitted.

Sanyal and Fox (62) indicated that economic analysis of the educational sector (as well as other segments of the public goods sector) has been impeded by the absence of market prices and the absence of price guided resource allocation. Systems analysis, according to Sanyal and Fox, provides a procedure to overcome these obstacles to economic analysis. There are four basic tools of systems analysis. They are: (1) Input-output analysis with its set of planning, programming and budgeting techniques which are used to generate efficient output sets; (2) efficiency analysis, where the arguments of the objective function and the signs of their first derivatives are known but the form of the objective function is unknown and the basic problem is the determination of efficient output sets, or, given a fixed output, the efficient input-output combinations are sought; (3) optimality analysis or mathematical programming, where the arguments and the form of the objective function and the input-output coefficients are known and the task is to achieve a maximum or minimum; and (4) simulation, where probabilistic values are assigned to the parameters (requirements) to yield a frequency distribution of resource requirements. Simulation can be used with any of the first three techniques. System parameters may be generated by current data, statistical estimation techniques or by normative values assigned by decision makers (62, pp. 11-14).

In the study at hand, the author utilized optimality analysis and statistical estimation techniques. Therefore, the literature review includes studies using these techniques (without regard to whether these studies dealt with macro-economic or micro-economic aspects of the education sector), and those studies concerned with public school education from the standpoint of the district, county, state, or region.

CHAPTER II. LITERATURE REVIEW

Resource Redistribution Effects

The provision of equity (educational opportunity), local tax relief, the stimulation of local expenditures and the promotion of efficiency in local expenditures are commonly stated as the goals of state aid plans to support primary and secondary education (7). Since state aid funds are raised by statewide sales, income and use taxes, and redistributed according to various formulas, state aid to local school districts represents a statewide (and sometimes countywide) redistribution of resources. Hence the goals of such a redistribution may be considered to be resource redistribution effects.

Most of the literature on the financing of public primary and secondary schools may be classified as either prescriptive or descriptive. That is, these writings make a case for broadening the educational tax base, for increasing the quantity of state aid or they describe the various forms of state revenue raising and state grant distribution plans. They also describe the theoretical redistribution effects of each grant type, the trends in total state support and trends in the utilization of each type of grant. For example, the report of the Committee for Economic Development presented a cogent argument for selective increases in state aid to education and for methods of distribution that theoretically equalize the ability of state school districts to support public education. A description of the importance of state aid across the United States was provided (18). Munse (55) presented the most current description of the types of distribution plans in effect. He also compared the number of plans and amounts disbursed under the various types of plans for the years 1953-54, 1957-58 and 1962-63 in order to ascertain trends in utilization of the alternative distribution methods. Good discussions of the theory of the redistribution effects of alternative grant distributions are to be found in Burkhead (13) and Benson (7). Empirical tests of the stimulation of local expenditure distribution effect of state aid programs have been reported by Davis (22) and Benson (7). A description of the educational tax base is available in Munse (54). Maxwell (49) presented a sound economic analysis of educational finance of state and local government and its relationship to financing. This aspect crosses into the broad area of public finance of which much has been written, but which will not be reviewed here. The investigation of the revenue side was not the main thrust of the research to be reported. There was little direct reference in the literature as to the effect of the form of finance on the distribution effects of the state grant plans. This was the only aspect of the revenue side considered in the empirical analysis.[1]

The Output Effects of State Aid Plans

A linear programming model was constructed in order to evaluate the output effects of alternative state aid plans. A sample of counties was

[1] Two sources indirectly related to this analysis, in that they emphasize the revenue raising side rather than the distribution side are Johns (34) and McLoone (53).

chosen to represent the state of Iowa. The various allocation plans were compared to an optimal distribution plan. The idea for this analysis was attributable to Benson's suggestion to allocate some portion of state aid on basis of meeting achievement goals. Linear programming has been used in the micro-economic and macro-economic analysis of the education sector. At the time of writing, it appears that the use of linear programming in a regional or state context is unique to the analysis presented in Chapter IV.

Beginning with Winkelmann's analysis of the teaching activities of a university department (87), several path breaking programming analyses pertinent to higher education have been performed. Plessner and Fox analyzed all of the basic university departmental activities (52, p. 29). B. Von Hoehenbalken and Fox utilized a quadratic objective function in a policy model to maximize the excellence of an academic department (81). McCamley demonstrated the feasibility of constructing a college programming model (52).

Hector Correa (21) proposed an approach to planning the educational curriculum that was presumably amenable to both university and to primary and secondary systems. Assuming that knowledge can be categorized into various branches, the branches were then divided into courses. Prerequisites for each course were determined. Correa specified an integer linear programming framework to analyze which courses and prerequisites should be offered. The procedure called for maximizing the product of each course times the benefits of each course summed over all courses offered subject to the quantity of instructional resources available (21, p. 689).

The determination of the benefit matrix was unspecified. The time horizon of the analysis was unclear. A high degree of stability of the curriculum over the time horizon was assumed. It seemed that it must be long enough for the group of incoming students that must meet requirements of a course of study to complete the program. Curriculum changes would mean having different groups of students in the system requiring different courses to complete their requirements. This would lead to a scheduling problem which could be approached by programming or nonoptimization analysis.

Optimization analysis has not been used extensively in the analysis of primary and secondary school systems.[1] Holtmann utilized a linear programming model to determine the relative value of various inputs to the local public school system. The analysis was short run in that it viewed the local school system as a profit maximizer with a fixed amount of inputs. The objective function to be maximized was the sum of the four types of student output (students terminating in grades nine through twelve) multiplied by the net additional lifetime income of the terminee. The cost per student was the variable cost associated with an additional year of schooling. There were teacher, classroom and student constraints. The students were allowed to drop out beginning in the ninth grade (32, p. 429). Holtmann calculated the maximum total product and then used the dual to get the value of the inputs. The slack inputs included teachers of certain subjects and students allowed to drop out. When dropping out was not permitted, the objective function decreased and the teacher constraint became binding (32, p. 436).

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[1]Examples of nonoptimizing analyses are Abt (2), Zabrowski (89) and Katzman (38).

Some limitations of this linear programming analysis are indicated by Holtmann. First, there are problems in the estimation of the value of the differential income to be earned by the various classes of terminees. There is much controversy as to the increase in income attributable to additional years of education (63). There is also the Weisbrod (83) option value to continue which is permitted by the completion of each year's education. Holtmann did not include the option values in his payoff matrix. There are problems in the specification of the input-output matrix. The paucity of information on the educational production function was alluded to in the Hoffmann article. He mentioned that there was little variation in staff student ratios because they are usually set at an administratively determined "appropriate value." (32, p. 432). Also, the problem of data on individuals versus averages was broached. In Holtmann's model, a graduate from any given grade represented the average curriculum that a student received in that particular grade (32, p. 433). It was clear that there was variance in the exposure of each student. Hence, the quality of education received by each student was not considered.

Koenigsberg (46) used linear programming techniques in his model of pupil assignment within school districts. Students were assigned to schools subject to restrictions on facilities and resources (e.g. a maximum travel time or a desired racial mixture) such that the objective function (e.g. the cost of total travel time) was minimized (46, p. 18). Koenigsberg indicated how bussing schemes, school location policies, educational parks and attendance boundaries could be evaluated on the basis of various efficiency measures such as cost or travel time (46, pp. 20-22).

The application of the linear programming approach to the allocation of resources within school districts was proposed by this author in the aforementioned unpublished paper (67). It was proposed that a factor analysis be employed to determine the relative weights of the various categories of staff inputs to the educational production function. In this case output was defined in terms of achievement test score. The instrument variables (inputs) were the number of staff personnel per student. The objective function to be minimized was the wage bill for staff salaries. The objective function was to be minimized subject to the constraints that the students achieve a minimum achievement score and that some minimum staff student ratios would be maintained (67, pp. 12-15). This linear programming analysis was not undertaken due to the problems of specification of the educational production function.[1] The factor analysis performed appears in Chapter V along with a discussion of the general problems of production function specification. Cohn presented a proposal for a school district linear programming analysis similar to that of Holtmann in that he suggested a maximizing framework except that he used achievement test scores rather than income (14, p. 53).

Correa's article on the allocation of national resources between general and vocational education was one of the more interesting macro-economic approaches to national educational planning (20). Correa suggested

[1]Specification of the production function was not necessary for the analysis presented in Chapter V. The coefficients assigned were based on those presently being used in the Iowa school districts.

maximizing an objective function consisting of students completing vocational or general education programs weighted by the respective yield of each kind of educational program. The nature of yield was unspecified, but presumably income was implied. The objective function was maximized subject to constraints that the number of general curriculum students multiplied by the cost per student plus the number of vocational curriculum students multiplied by the cost per student is equal to the total stock of education resources. Correa noted that the number of student graduating the vocational program could be constrained by the number or projected amount of job vacancies or by making income from vocational occupations an inverse function of the supply of persons for technical oriented occupations. He raised the questions of economies of scale and of the effect of the increased supply of generally educated personnel on their value. However, he argued that these obstacles to analyzing problems were not insurmountable (20, pp. 107-113).

Corazzini and Bartell's critique of Correa's analysis revealed some of the difficulties of the application of the programming framework to the education sector. They argued that Correa did not distinguish between private and social costs, and private yields and social yields. They also argued that public costs and private benefits were being compared in the programming framework constructed by Correa. Hence, there was a need to segregate and to account for benefits to the individual, the society and for joint returns. They felt that the division of the education sector into only two broad categories was unrealistic because there was substantial diversity within the two categories as specified and they should not be treated as two homogeneous parts. They objected to the implicit assumption that vocational and general education were independent of one another with respect to costs and yields and to the assumption that all students were equally capable of absorbing all types of either vocational or general education. They found fault with Correa's treatment of economies and diseconomies of scale. Corrazzini and Bartell felt Correa should have considered the opportunity costs of diverting resources to the educational sector from other sectors (19). Their final criticism was that some kind of transfer activity should be specified to shift the possible slack resources in the optimum solution out of the educational sector. The objection to fixity of resource supply and to constant input-output coefficients was overcome to a certain extent in Bowles' paper. The objection to the treatment of the educational sector in isolation was considered in the Adelman (3) and Bowles (10) papers.[1]

Bowles was concerned with allocation of resources both to and within the educational sector such that the contribution of the educational system to future national income was maximized. "The constraint equations define what can be called an intertemporal production possibility set for the educational system" (10, p. 191). The educational system activities of his system were primary education, secondary education, higher education, teacher training, technical and vocational education. The activities used

[1]These were cited in McCamley (52) as examples of macro-economic education models.

inputs to transform both intermediate and raw materials into finished products. Flows of students and teachers through time tied the activities together. The effects of changing educational techniques were investigated (10, p. 200). Also, the optimal total resource use by the educational sector via the use of shadow prices (10, p. 209) and the effect of importing educated labor was explored (10, p. 212). Bowles indicated some limitations of his model. These were similar to the limitations of Correa's programming model as expressed by Corazzini and Bartell. However, Bowles stated that these types of shortcomings were balanced by the use of the model to make "explicit the complicated interrelations within the educational system with (such that) it allowed the investigation of the direct and indirect effects of a multiplicity of concrete policy choices" (10, p. 216).

Adelman considered the educational system as one of many sectors of a planning model for Argentina. Her depiction of the educational sector was similar to that of Bowles. She introduced several objective functions (e.g., to maximize the rate of growth and to maximize the employment rate) (3). Some portions of her results were, as pointed out in Cohn (14, p. 42), unrealistic. This was attributed to her estimation of some of the payoff coefficients. Her estimation difficulty was illustrative of a problem typical of the application of any economic model to the real world.[1]

Specification of the Relation between Educational Output and Input

Most of the problems with respect to the application of programming to problems of the educational sector originate with the problem of

[1]Recent macro-economic educational planning studies have been conducted by LeVasseur (47), Williams (86) and Freytag (27).

specification of a production function for education. The output of education has been measured in terms of number of student terminees from the educational system, income earned by the students, placement of students and achievement or educational as development measured by standard tests. This list neglects the measures of the process of education (evaluation of costs of inputs, Time Scale, Growing Edge) that have been used as proxies for educational output (6, 63, 84, 85). It can be argued that scores on examinations designed to measure past performance are good indicators of future performance (78). It can be further argued that such tests are more closely related to school policy variables because they are, to some extent at least, a measure of what the school contributes to the student's development (48, 33). Because of the imperfections of the market place and the problems of interpreting relations between income and education, it was assumed in Chapter IV that the Iowa Test of Educational Development would be a more appropriate measure of educational output in a regional context.

In the macro-economic linear programming models presented above (3, 10), the input coefficients were estimated on the basis of historical and current data on teacher student rations and other inputs. The payoff matrix was estimated exogeneously. In a similar manner, the current input coefficients (expenditures) and output coefficients (achievement test scores) were used in the state aid distribution model described in Chapter IV. In other words, the actual educational production function relating educational output to educational input was left implicit in this study. Bowles and Adelman did not specify any explicit education production function relating their education outputs and inputs. Thus, in all cases sets of points (output-input combinations) were utilized that may or may not have been elements of the appropriate production function.

By late 1969, Welch alone had attempted to specify an explicit relationship between educational inputs (total current expenditure per pupil, average salary per member of instructional staff, members of staff per 100 pupils and enrollment per secondary school and income. In all cases his regression equations at least explained 90 percent of the variance in income. When the last three mentioned school variables were included in the regression, the partial regression coefficients for all of the variables except the student teacher ratio were positive. This positive relation found between output (income, in this case) and teacher's salaries is common to the findings of most of the studies cited. When Welch considered current expenditure alone (that is, as the only school input), it carried a negative coefficient. When the effects of any two of the other three variables were accounted for, the coefficient became positive. Subsequent analyses performed by Welch by means of various combinations of regression runs, indicated that the teacher-pupil ratio had an unexpected effect. The partial derivative of output with respect to the teacher pupil ratio was found to be negative in each run (85, pp. 390-392).

This is contrary to the prevailing notion that educational output increases as the number of teachers per student is increased. Since per pupil instructional expenditures equals the product of total salaries paid divided by the number of teachers multipled by the teacher-pupil ratio, it can be seen that the effect of current expenditures (instructional expenditures account for the largest portion of current expenditures) is the resultant of two conflicting forces. The teacher-pupil ratio exerts a negative effect and the salary expenditure exerts a positive effect.

Often, in the studies where test scores are used as the measure of educational output, the teacher pupil effect appears to swamp the salary effect. This swamping causes the partial derivative of output with respect to expenditures to be positive.

Welch hypothesized that the effects of teacher-pupil ratios depend on the size of the school. Higher teacher pupil ratios can be associated with small schools and a diversified teaching load. Hence the teacher pupil ratio may be a proxy for teacher specialization and the increased productivity associated with it. Hence, larger schools may have larger classes than smaller schools, (that is larger schools may have lower teacher pupil ratios), but each individual student achieves more in the larger class of the big school being led by specialists than a student in the smaller classes of the smaller school where specialists are absent (85, pp. 387-391).

Studies reported in Chapter V included those in which standardized tests and educational process measures (such as the Growing Edge) have been used as proxies for the output of the education sector. The findings of recent studies conducted by Cohn (14; 15), Gavin and Spitzer (28), Kiesling (43; 44), Mayeske (50), Thomas (77) and several studies (including Ayer's) conducted prior to 1958 as summarized by Ross (61) are presented.

CHAPTER III. AN ANALYSIS OF THE RESOURCE DISTRIBUTION EFFECTS OF STATE SCHOOL AID PLANS

Introduction

Recent research into the economics of education has focused upon either the macro-economic or micro-economic point of view. Within the macro-economic framework, much of the work has been concerned with the underinvestment in education arising from the disassociation of benefits and costs. The work of Schultz (63), and Weisbrod (83; 84) represent this approach. In the micro-economic framework, the investigation has centered around the determination of educational production functions (43; 44), optimal size of schools (15), least cost combinations and system-analytic costing of alternative education programs (65).

Less emphasis has been placed on the study of resource allocation to education on the regional, state, county or district level. In recent years local governments have experienced increasing difficulty in financing public schools. This is due in part to the fact local community leaders feel the upper bound of revenue from local property taxes is close at hand or has been reached. State funds have had to bear a larger part of the burden of financing public education.

It has been stated that alternative forms of state grants in aid to local districts have differential resource redistribution effects. For this reason, in the era of increasing share of state funds in the local school budget, the choice of plan is cause for public concern. The resource redistribution effects of such plans (often referred to as goals in the literature of educational finance) are equity, property tax relief, efficiency of local expenditures and stimulation of local spending on education (7, p. 105). Equity refers to individual educational needs relative to the distribution of educational services and to the distribution of the burden of paying for them. Equity is sometimes referred to as equality of educational opportunity (17). The stimulatory effect of federal grants in aid on state and local spending has been investigated by Davis (22) and others. Only Benson and Kelly did empirical research on the stimulation effects of a particular form of state aid. Their findings were reported in Benson (7, p. 189).

The purpose of the research reported in this study is to disaggregate educational data so as to examine public education at county and local levels to test the hypotheses that (1) the alternative forms in which state school aid can be allocated have differential redistribution effects dependent upon the economic-demographic environment, and (2) that the redistribution effects of the alternative allocation methods are affected by the form of finance employed by the state to fund school aid. The efficiency and stimulation effects are not considered in the empirical analysis. Preceding the empirical analyses is a discussion of alternative state aid allocation plans, an analysis of their geographic distribution and a description of trends in plan use.

Alternative State Aid Allocation Plans

Burkhead (13, p. 210) presented the following taxonomy of state aid programs which will be used in this study. His three major headings are as follows:

(1) Use of proceeds, (2) resources measure and (3) needs measure. The first classification is divided into (a) general purpose and (b) special purpose. When the resources measure (2) is used the aid can be equalizing or nonequalizing. The needs measure (3) can be based upon either (A) percent of local expenditures or (B) unit costs of education. Either (1) fixed or (2) variable unit costs may be considered.

Classification of plans

In order to obtain state funds under a plan, it may only be necessary for a district to state general purposes. Sometimes special or specific expenditure plans are required to be stated before funds are obtained from the state. A first classification characteristic of a state grant program is whether the aid is distributed either with or without restrictions on the use of the funds. Many state programs consist of combinations of general and special purpose grants. For example, some states distribute the largest share in the form of unrestricted or general aid and smaller shares are earmarked for vocational education, transportation and handicapped children. The second characteristic of classification refers to the objective of equality of educational opportunity. Equality of educational opportunity can be defined as making some minimum level of educational services available to all students in the state regardless of the ability of the student's district to obtain these services, without reducing all to this minimum level (7, p. 155).

The third characteristic which Burkhead (13, p. 210) distinguished was whether the state grant policy accounts for local effort, that is the amount the local district is spending relative to its resources. Implicit here is the idea that no district should be unduly burdened while providing educational services. The key consideration under this category is the formula by which local effort is measured. Aid can be determined by comparing expenditures from local sources to a standard or fixed level. It can vary with the absolute amount of local effort put forth or be proportional to the district's relative wealth. It can be noted that the specific distribution formulas are combinations of the items under the second and third categories of the taxonomy. Based on notation adopted from Benson (7) and Burkhead (13) these combinations may be generalized as shown in Table 1. The terms are defined in Table 2. In this analysis which borrows heavily from Benson (7) and Burkhead (13), emphasis is on distribution of general aid.

Table 1. Combinations of resource and needs measures in state aid distribution formulas

	Equalizing	Nonequalizing
Unit costs of education		
Fixed	$A_{i1}=N_iU-r_iW_i$	$A_{i2}=N_id_ia_{ij}$
Variable	$A_{i3}=N_iU_i-r_iW_i$	$A_{i4}=N_iU_i$
Percent of local expenditures	$A_{i5}=1-[(X)(W_i/W)]E_i$	$A_{i6}=(1-X)E_i$

Table 2. Definition of symbols of state aid formulas of Table 1

Symbol	Definition
A_{ij}	State aid paid to the ith district under the jth plan
N_i	Number of students in the ith district
r_i	Tax rate of the ith district
W_i	Tax base of the ith district, standardized, set equal to minimum or maximum
U	Minimum level of expenditure per student
a_i	Specified amount of aid per student
d_i	Number of days in session of the ith district
E_i	Total reimbursable expenditures in the ith district
X	Approximates proportion of local share of state expenditures on education
W	Total wealth in state

Comments on the alternative plans

The impact of A_{i2}, fixed unit, nonequalizing appears rather straight forward or neutral. Sometimes, a_{ij}, the amount per pupil varies among elementary, high school and community college students. N_i, the number of students, can be measured by either average daily attendance, average daily membership (the mean of average daily attendance plus average daily absences) or the census of 5 to 21 year olds. This last measure is favored by districts having a substantial percent of private school attendance.

This A_{12} system is neutral only in states with uniform population density. In reality, it is possible to argue that this distribution plan is not neutral in the sense that it favors suburban areas. This favorable impact is based on the phenomena of scale effects which are occurring in surburban areas (15, pp. 96-108). Up to a certain enrollment the exact level of which varies by region, the cost per student decreases. Therefore, receiving a fixed amount per student when costs are decreasing eases the burden on local revenue sources. Conversely, receiving a fixed amount when cost per student is rising increases the local burden. This is often the case in rural areas experiencing out-migration. This out-migration forces these rural areas to start moving back (up to the left) on their cost curves.

However, there are diseconomies that occur after a certain enrollment and cause either upward movements (to the right) or shifts of the cost curve. The cost per student in urban areas can rise above the basic minimum or foundation set by the state because of higher teacher salaries, higher site costs, attendance imbalances leading to underutilization of resources caused by population shifts, high pupil turnover, and changing composition of student population, the nature of which requires special programs. Vocational programs requiring extensive equipment are particularly expensive (13, p. 231). Of course, urban areas do not have the large transportation costs. (This statement would have to be modified, depending on extent of crossbussing.) Suburban areas are not burdened by the problems caused by extremes of size, although many of them are experiencing the need for rapid expansion of their physical plants. It has been argued that by virtue of its inflexibility, the fixed unit, nonequalizing A_{i2} plans fail to meet any of the distribution objectives of state grants.

Expenditures are not stimulated because, as previously discussed, many districts are already spending amounts above the foundation level (a_{ij}). Hence, the full burden of any additional expenditures falls on local resources. Because of property taxes the repressive nature are the mainstay of local finance, this A_{i2} method is deficient on equity grounds. It can be argued that fixed unit, nonequalizing distribution programs conform to the efficiency and tax relief criteria. This is only to the extent that gains from the use of modern technology in the educational process are in the form of either tax savings or improvements in educational services and that these gains accrue directly to the residents of the locality.

Under fixed unit equalizing, A_{i1}, aid is the difference between some state determined student expenditure norm ("U ") and the amount the district can raise (local effort). Usually, there is a minimum local tax rate (r_i). This is set equal to that prevailing in the wealthiest district. The rationale is that no district should have a tax rate greater than the lowest rate compatible with quality education. That is, the combination of educational, expenditure and effort of the richest district is taken as standard. This does not imply that all districts should spend the same as the richest district, but each district should be enabled to afford some minimum level of educational opportunities represented by "U" (13, p. 212). This of course presumes that a dollar spent on education yields the same benefits all over the state, which fails to hold if economies or diseconomies of scale prevail.

The method of wealth measurement becomes a factor here. The modal measure of wealth under system A_{i1} is real property. Even if there were not the plethora of assessment problems, it is questionable whether real property value can be taken as a perfect correlate of wealth in these days when size (value) of home or other holdings of real property may not be a result of how one earns a living. It is often a measure of family need and not closely related to one's wealth. Hence, the proportion of the intangible assets held may be a better measure of wealth. Holdings of these assets are increasing. Further, an increasing proportion of income accrues to professional services rendered and the provision of these is not land intensive. The use of real property as both the major portion of the tax base as well as the measure of wealth seems to represent an anachronism perpetuated because it is expedient. The use of income, recognizing the problems of its specification as part of the wealth measure, albeit not taxed by the district, ameliorates this problem of measuring ability to pay.

Under A_{i1} the net grant is somewhat proportionate to district ability to pay, providing the state taxes have equal incidence on all districts and that state and local taxes have the same tax base (13, p. 213). These conditions are rarely met. Most states depend on the sales tax (7, p. 106), both of which are considered regressive. Hence the fixed unit equalizing grant is lacking on equity grounds.

The variable unit equalizing plans A_{i3}, differs from the fixed unit approach in that U, the foundation level is allowed to vary within certain bounds. The symbol "U" is subscripted to denote local variation. The U_i is not what is actually spent, but it is generated for state aid computation purposes. In this determination, the district is credited for the quality of its educational inputs. For example, given a minimum teacher pupil ratio, the district is credited with a certain amount per teacher depending on his experience and training. It is argued that plans following the outline of A_{i3} do not differ from fixed unit plans with respect to the four goals for state grants for equity (13, p. 219). The contention is that the variable U in A_{i3} is in effect a subsidy for the purchase of higher quality (and more expensive) educational resources and that this subsidy accrues to the wealthier districts able to take advantage of it.

In a study of variables determining the expenditure policies of schools, Davis postulates that the existence of Pennsylvania's combination of fixed and variable unit equalizing plan would strengthen the positive effect of property value on school expenditure due to the slackening of the local budget constraint (22, p. 99). He did not directly test the influence of the Pennsylvania state aid distribution plan, although the regression coefficients of property did have the postulated positive sign (22, pp. 101-107).

Percentage equalizing plans, A_{i5}, are the newest, least commonly used of the types heretofore discussed. The basic idea is that the state grant depends on a given percent weighted by the district's relative ability to pay and on its own reimbursable expenditures (13, p. 220). There are alternate definitions of ability to pay (W_i/W) and of reimbursable (E_i) expenditures as well as in the weighted percent value determining local share (X).

Ability to pay can be converted to a per student basis; income, real and personal property, can be included in weighting factor. E_i is usually expenditures of the previous year, just as is the U_i associated with A_{i3}. In the year of initiation, especially, the issue of whether to include the previous year's state aid as well as federal assistance in total local spending becomes important. It can be noted that when (W_i/W) equals one, (1-X) becomes the state's proportionate share of district expenditures.

From the standpoint of equalization, percentage equalizing grants (A_{i5}) are considered most effective. The percentage equalizing plan permits local determinations of needs. In contrast to all of the above plans under the variable unit equalizing plan, A_{i3}, state officials determine the normative value of resources allocated to education. Despite the necessary expenditure differences caused by local factors, each district will be financing its expenditures at about the same rate (13, p. 220).

Operational decisions can distort the theoretical elegance of this approach. If X is too low, the A_i to wealthy districts will become negative. Unless the district is forced to contribute to a state equalization (that is, to receive negative aid) the equalizing function becomes distorted. This happens both when negatives are regarded as zero and when a lower (positive) bound for all districts in the state is determined. Local tax relief is theoretically built into the plan. The effect on the stimulation of local expenditures as well as on efficiency of local expenditures depends on whether local tax rates or the total local tax bill is the overriding consideration. Inefficiencies as well as other increases in local expenditures are shared rather than borne entirely by the district.

Contrariwise, the bill for educational expenditures is highly visible which can dampen expenditure and encourage better utilization of resources. Benson and Kelly concluded that the institution of a percentage equalizing plan in Rhode Island did not stimulate local expenditure, but did provide tax relief and equalize by differential increasing of the state's share (7, p. 189).

In summary, plans can theoretically have differential effects with respect to expenditure stimulation, equity, (equality of opportunity), efficiency and tax relief. It is thought that a plan's outcome besides being tempered by community attitudes toward education, depends on the economic structure of the state. This includes income distribution, nature of industries, location of industries, population distribution (urban versus rural) and the tax system. Also, after considering all these factors, the administration of the plan decides its effectiveness. Furthermore, even if some balance of the above distribution effects is achieved, there is no guarantee that district expenditures per student are arranged so that educational output is maximized (13, p. 235).

Geographic distribution of state aid plans

With respect to the geographical distribution of the various state grants, Burkhead indicated that of the 389 programs administered in the United States in 1957-58, 284 were of the special purpose type, but 82 percent of the funds were distributed on a non-restrictive or general basis (13, p. 208). Of these 105 general purpose grants, slightly less than half were distributed on an equalizing basis with the fixed unit basis the most prevalent. He also commented that there was considerable regional polarization with respect to the types of general purpose equalizing grants employed. The fixed method predominated in New England, the Mideastern Great Lakes and Far Western states while the variable method was found most often used in the Southeastern, Southwestern and the Rocky Mountain states. Only one state, Wisconsin, employed the percentage equalizing method.

Trends in plan use

Table 3 was generated from classification of 1962-3 state grant descriptions provided by Munse[1] (55, pp. 13-93). In Table 4, these data were summarized and superimposed on Burkhead's table. A basic change in the structure of state aid can be observed from inspection of these two tables. This change is the increased use of the percentage equalizing grant. Virtually the entire Mideast region has turned to this type of distribution. It can also be observed that the number of equalizing plans has decreased. However, these plans have increased in importance in that the percent of total state funds distributed in this manner has risen from

[1]The application of taxonomic schemes to real world phenomena is not always straight forward. There are often border line situations.

This was especially true with respect to classifying certain equalizing plans as fixed or variable. Many states had a different foundation amount for primary and secondary schools. These amounts were sometimes further adjusted for size and location (urban-rural) of the district. In order to generate data comparable to Burkhead's, foundation plans were classified fixed unless they had adjustments to account for local choice of educational quality, such as training level of teachers hired and the presence of specialized professional personnel.

Table 3. Estimated number of general purpose state grants distributed as equalizing, nonequalizing, unit and percent, for 1962-63[a]

State	Equalizing				Nonequalizing			
	Unit	Fixed	Variable	Percent	Unit	Fixed	Variable	Percent
Alabama			1			1		
Alaska						1	1	
Arizona						1		
Arkansas		1				1		
California		1				1		
Colorado		1				1		
Connecticut							1	
Delaware							1	
Florida			1			2		
Georgia			1					
Hawaii							1	
Idaho			1					
Illinois		1				2		
Indiana		1						
Iowa		1				2		
Kansas		2				3	1	
Kentucky			1					
Louisiana			1			1		
Maine		1		1				
Maryland			1			3		
Massachusetts		1					1	
Michigan		1				1	1	
Minnesota		1				3		
Mississippi			1			1	1	
Missouri		1				3		
Montana		1				2		
Nebraska						3		
Nevada		1						
New Hampshire		2						
New Jersey		1				1		
New Mexico						2		
New York				2				
North Carolina							1	
North Dakota		1				1		
Ohio			1					
Oklahoma			1			3		

[a]Calculated from 55, pp. 13-111.

Table 3. (Continued)

State	Equalizing Unit	Equalizing Fixed	Equalizing Variable	Equalizing Percent	Nonequalizing Unit	Nonequalizing Fixed	Nonequalizing Variable	Nonequalizing Percent
Oregon		1				3		
Pennsylvania				1				
Rhode Island				1				
South Carolina						1	1	
South Dakota						1	1	
Texas			1			1		
Utah		1						
Vermont						1		
Virginia		1					1	
Washington		1	1			1		
West Virginia			1				1	
Wisconsin				1		1		
Wyoming		1				1		
Totals		24	14	6		49	13	

Table 4. Distribution of general purpose grants 1957-58, 1962-63[a]

	1957-58	1962-63
Equalizing		
Unit	48	38
Fixed	28	24
Variable	20	14
Percent	1	6
Subtotal	49	44
Nonequalizing		
Unit	56	62
Fixed	51	49
Variable	5	13
Percent	0	0
Subtotal	56	62
Total	105	106

[a]Calculated from Table 3 and (7, p. 210).

51.1 percent in 1957-58 to 57.8 percent in 1962-63 (55, p. 112).

Consideration for Empirical Analysis of Economic Effects of State Aid Plans

Choice of four representative plans

Thus far, various methods of allocating state aid have been defined, the theoretical implications with respect to the resource redistribution aspects of expenditure stimulation, equity, efficiency, and the provisions of tax relief have been discussed. The actual effects of these plans, as has been mentioned, depend on the interaction of the administration and the economic-demographic setting. In order to explore further the implications of these plans, the effects of four prototypes on six counties in the state of Iowa were analyzed. The plans used are described in this section.

Plan 1 is of the fixed, nonequalizing type (A_{i1}) where a_{ij} is the amount of aid to the ith district for jth level of student, where j=1,2,3 representing grades K-8, 9-12, 13-14, equals \$.17, \$.20, and \$1.00 respectively. N_{ij} equals the average daily membership of the jth class;[1] d_i equals the number of days in session up to a maximum of 180.

This plan along with minor supplementary and specific aids was used in Iowa until the middle of the 1967-68 school year (68, pp. 134-142). Since the major purpose was to isolate the effects of various distributional approaches, the absence of the other plans was assumed. Each district

[1] In the original formulation average daily attendance was used in place of average daily membership.

was assumed to receive only general aid in the particular form being discussed.

Plan 2 is of the percent equalizing type (A_{i5}). In this version the general formula is expanded to include two forms of wealth, income (Y) and property (P), and converted to a per student basis. Also, there is a feature that is intended to facilitate county-wide as well as state-wide equalization (69, pp. 76-86). This formula appears below. The county-wide equalization is based on the district share of the county tax equalization fund (C_i), which consists of 40 percent of the sum of total tax askings in the county plus the state income tax collected in the county. The district receives (pays out) this amount to the county fund. This amount is removed from the estimate of reimbursable expenditures (E_i) before the state aid is distributed.

$$A_i = (1 - .25 \frac{(.7P_i + .3Y_i)/N_i}{(.7P + .3Y)/N} \quad (E_i - C_i))$$

$$C_i = ((.4T_j + .4IT_j)/N_j)N_i$$

P_i = market value of real property (including utilities) in district i

P = market value of real property (including utilities) in state

Y_i = gross income earned in district i

Y = gross income in the state

N_i = (ADM + Census of 5-21 year olds)/2 for the ith district

N = (ADM + Census of 5-21 year olds)/2 for the state

E_i = reimbursable expenditures of ith district

C_i = aid to ith district from county

T_j = total property tax askings in the county; the sum of all the district askings

IT_j = total of state income tax collected in county j.

It should be noted that since T_j depends on how much state aid the districts will receive, it is necessary to use state aid of the year before calculating state aid for the current year.[1]

The third distribution plan used is a more traditional version of A_{i5}. Under this plan, wealth (W) is the market value of real property converted to a per student basis; the percent of local share weighted by the index of relative wealth was .6 and local expenditures (E_i) are defined above. Ten and 70 percent are the lower and upper bounds respectively for the state share in any district.

The last prototype is a combination of A_{i1}, the fixed equalizing plan and A_{i2}, the fixed nonequalizing plan. All districts are to receive the difference between the basic state grant of $80 per pupil plus 8 mills tax on the market value of district real property (including utilities) per student and a $550 foundation plan. The formula below restates this plan in terms of total aid to the district. Note the aid consists of two components, district basic aid and a district increment.[2]

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[1]Once the plan is in operation, year t's aid depends on t-1's expenditures and local taxing, both of which depend on that year's aid. Since this program was initiated in the middle of the school year, an iterative process was employed to calculate the necessary parameters.

[2]This combination and the values of the parameters were suggested during a conversation with Hartsel Perry, Consultant to State of Iowa, Department of Public Instruction.

DBA_i = district basic aid

$= \$80\ N_i$

DI_i = district increment

$= \$550N_i - [(8\text{ mills})W_i + DBA_i]$

$A_i = DI_i + DBA_i$

Selection of sample

In order to measure the distribution effects of state aid plans, a sample of six counties was chosen from the state of Iowa. The choice of counties was governed by the desire to include economic-demographic configurations typical to a plains state such as Iowa, and simultaneously to maintain geographic balance. These geographic considerations are particularly important in rural areas. Inferences based on these data may be made to other counties or states only to the extent that equivalent situations exist. The counties identified by letter and their characteristics are listed below:

A - school districts in a standard metropolitan statistical area surrounded by rural districts; 63,503 student population located in center of Iowa, subsequently referred to as "urban county."

B - school districts divided approximately equal between rural and urban; 28,325 student population located in northcentral Iowa; subsequently referred to as "mixed county."

C - same as B; 35,690 student population located in eastcentral Iowa.

D - rural district; 2,742 student population located in northwest; subsequently referred to as "rural county;" counties E and F are members of this set.

E - rural district; 4,328 student population located in southeast Iowa.

F - rural district; located in northcentral Iowa.

Assumptions and calculations

The analysis was predicated on the assumption that the above described six counties made up the entire state and that the state had appropriated the amount necessary to fully fund the second plan. This was because the type of distribution was the variable of interest. Full funding of the other plans would generate a different total for each plan. Each district's share under Plans 1, 3, 4 was determined and its respective percent share relative to the state was calculated. This percentage for each district was multiplied by the base appropriation. The result was a set of adjusted or standardized district shares for each plan, based on 1967-68 Iowa data provided by the Department of Public Instruction (71) and Office of the Comptroller, State of Iowa (76). Total local district expenditures budgeted for the year were reduced by the difference between total outside aid and expected general aid under the previously used fixed non-equalizing plan. This yielded the amount the district would have to finance if there were not any general aid. This term was defined as general aid reimbursable expenditures. The general aid calculated from each plan was subtracted from general aid reimbursable expenditures. This amount was to be financed by the local property base defined as the market value of local wealth. Local wealth was defined as real, personal and utility owned property. The conversion from assessed to market value was based on ratios between sales and assessed values calculated annually by the state. Local tax askings divided by the local tax base resulted in the local millages.

Effect of Plan Distribution on Millage

Since property taxes make up the bulk of the funds financing schools, the analysis of the effects of the various forms of state grants resolved itself into the following diagnostic hypotheses.

1. What is the effect of the various plans on local millage? (tax relief)

2. Are the millage effects different from district to district and region to region? (equity)

Description of plans effect on millage

For each of the four plans, the mean and coefficient of variation of the local millages was computed for each county and for the six counties taken together (Table 5).

Of the six counties combined, Plan 2 has the lowest average millage, 48.769, but the highest amount of variation. The lowest millage would be expected because income taxes carry a portion of the burden of local education expenditure under this formula. For example, in the urban district school districts revenue from income taxes are six percent of the local contribution for education.[1] Among the instances where no income taxes are available, Plan 1 yields the lowest overall millage but highest variation. Among the urban and mixed counties A, B and C, the lowest average millage (51.496) is achieved under Plan 1 in county B and the highest average (63.709) as well

[1]From a theoretical standpoint, it might have been advantageous to compare results of a proportional sharing plan that included income in the calculation of wealth, but had no tax receipts from income or county equalization feature.

Table 5. Millage for alternative plans (1967-68 Iowa data)[a]

County		Plan 1	Plan 2	Plan 3	Plan 4
6 Cty	$\bar{X}$[b]	54.326	48.769	56.220	56.189
	co.v[c]	0.141	0.182	0.113	0.114
A	$\bar{X}$	63.709	58.839	61.889	63.311
	co.v	0.262	0.166	0.213	0.226
B	$\bar{X}$	51.406	52.345	53.160	51.482
	co.v	0.081	0.155	0.066	0.068
C	$\bar{X}$	62.397	55.951	61.045	61.907
	co.v	0.139	0.094	0.110	0.119
D	$\bar{X}$	53.386	34.452	58.667	58.433
	co.v	0.768	0.180	0.700	0.708
E	$\bar{X}$	51.899	49.574	57.625	55.490
	co.v	0.196	0.175	0.160	0.188
F	$\bar{X}$	43.161	40.961	44.933	46.511
	co.v	0.081	0.102	0.109	0.092

[a]Calculated from 71 and 76.

[b]$\bar{X}$ = mean.

[c]co.v = coefficient of variation.

as the greatest coefficient of variation (.262) is in county A. The lowest average millage (40.961) is achieved in rural county D (35.452) under Plan 2. The highest millage is encountered under Plan 3 in county D (58.677). The most rural relative millage variation is 0.708 and occurs under Plan 4 in county D.

Statistical analysis of the plan effect on millage

In order to generalize the results, a two-way analysis of variance was performed on the county averages (i.e., six counties taken together) and on each of the individual counties (24, p. 156). Hence millages were classified by plan and geographical unit (county or district). See Table 6. The analysis of variance indicates the significance of the plan and the geographical unit on millage. Based on the six county averages both the plan and the county have a significant effect on millages.[1] On a district basis, the plan effect on millage is significant in three out of the six counties (the urban county, one of the mixed and one out of the three rural counties).

Table 6. Effect of plans (columns) and counties or districts (rows) on millage as measured by F values (1967-68 Iowa data)

Geographic area	Plan effect	County or district effect
6 county	4.638*	10.651*
County A	3.271*	42.541*
County B	0.174	2.476
County C	6.158*	9.843*
County D	1.813	9.879
County E	16.063*	112.596*
County F	1.460	3.16

*Significant at five percent level.

[1]In all cases the 5 percent level of significance was used. In one case out of twenty an effect would be judged significant when it was not. Because of political difficulties of changing the form of state grants, it was decided that it was important to be at least 95 percent confident when effects were judged significant. In other words the consequences of a higher type two error (accepting the absence of any plan effect) was less costly than a higher type one error.

A statistical comparison analysis (88, p. 658) to determine which plan or plans is responsible for the significant difference among the means yielded interesting results. As one expected, the millage levy under Plan 2 (in which there are other non-district funds besides direct state aid, was significantly different from millage levies under Plans 1, 3 and 4. This was true for the individual counties as well as for the six county averages.[1]

However, in three of the four individual counties, there was no significant difference between the millages under the other three plans when compared on the two by two basis.

Plan 4 and Plan 1 had dissimilar effects in rural county E. Since the plan effect was not significant in two other counties, this meant that the differential effects of these plans were not large enough in the sample counties to suggest differential effects in the population taken as a whole or stratified by economic demographic characteristics.[2] This was a rather interesting finding given the dissimilar methods of calculation and theory discussed above.[3] Hence, these three plans can be said to have similar property tax relief effects.[4]

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[1]The difference between Plan 2 and Plan 3 in urban county A was just barely significant.

[2]Perhaps this finding would be unique to the Iowa sample. If so, it might arise because there was not a large difference between the areas considered rural or urban.

[3]It should be recalled that the first plan is of the fixed, non-equalizing type (A_{i2}), the second of the fixed equalizing and the third of the variable equalizing type (A_{i5}).

[4]There was no longer a close correlation between income and property. Income is not included in wealth measures used in Plans 3 and 4. This is consistent with the traditional versions of these plans. However, the distribution of income and property wealth in urban versus rural counties could have contributed to the nonsignificant plan effects on millage.

Characteristics of the six county sample

Because of the unique effects of the 1967-68 Iowa variable unit equalizing plan, it was believed desirable to make a more detailed comparison of this plan versus alternatives. It should be recognized that given the statistical results presented above, generalization of the findings discussed below would be hazardous. The following discussion is based on Table 7.

Since Plan 1was typical of many types currently in use, it was used as a base to further investigate the millage effects of the other plans. The local millage under Plan 1 was subtracted from the local millage applicable to Plans 2 through 4. A negative outcome represent a decrease in millage under the respective plan. In terms of both property tax relief and equity, it was interesting to observe the effect on the largest district in the county. Assuming a uniform distribution of wealth, the largest districts in urban or mixed counties tended to have the heaviest burdens.[1]

In the urban county, 78 percent of the districts including the dominant district had their millage reduced by the imposition of Plan 2 subsequent to Plan 1. Approximately the same proportion of mixed and rural districts enjoy millage decreases (69 percent versus 74 percent), but while all of the mixed dominant districts suffer millage increases, none of the rural dominant did.

[1]This situation was compounded by the fact that the gap between actual and an assessed value of real property is less in the central city than in rural areas and in some cases the gap might be negative.

Under Plan 3, less than half of all districts showed decreases, one of which is the dominant district in the urban area. Only one rural dominant district showed a decrease. Eighty-six percent of the rural districts have increased millages, compared with only 50 percent of the mixed and urban districts.

None of the dominant districts showed any decrease when Plan 4 was substituted for Plan 1. Approximately two-thirds of the urban districts show decreases, but less than half of the other districts did.

Based on the above results Plan 2 is of greatest benefit for all when viewed from a tax relief or millage equalizing basis. Plan 3 can be of some help to large urban districts, but not to moderately large mixed districts. Plan 4 would be detrimental to all relatively large districts and to over half of the other districts from a tax relief standpoint. If these are not the poorest district perhaps there are equity considerations that ameliorate these results.

Plan 2, as was mentioned, has a rather unique county equalization feature. The district subsidy from or to the county was computed on a per student basis for each county. This gain (loss)was the difference between what it pays in to the county equalization fund and its receipts from the fund. The qualitative results are summarized in Table 8, which shows the proportion of districts gaining in each county, and whether the dominant district gains or loses with the imposition of Plan 2. The dominant district is the district with the largest amount of students, and presumably the one with the heaviest burden. In the urban county, the dominant

Table 7. Analysis of the change in millage structure change using Plan 1 as a base (1967-68 Iowa data)[a]

Plan		County	All districts				Dominant district	
			Decrease		Increase			
			Number	Percent	Number	Percent	Decrease	Increase
2	Urban	A	7	78	2	22	X	
		B	3	60	2	40		X
		C	8	73	3	27		X
	Mixed		11	69	5	31		
		D	4	67	2	33	X	
		E	3	75	1	25	X	
		F	3	100	0	0	X	
	Rural		10	77	3	23		
	Total		28	74	10	26		
3	Urban	A	5	55	4	45	X	
		B	1	20	4	80		X
		C	7	64	4	36		X
	Mixed		8	50	8	50		
		D	0	0	6	100		X
		E	0	0	4	100		X
		F	2	67	1	33	X	
	Rural		2	14	11	86		
	Total		15	39	23	61		
4	Urban	A	6	67	3	33		X
		B	1	20	4	80		X
		C	5	46	6	54		X
	Mixed		6	37	10	63		
		D	0	0	6	100		X
		E	0	0	4	100		X
		F	0	0	3	100		X
	Rural		0	0	13	100		
	Total		12	46	26	54		

[a]Calculated from 72 and 76.

district gains. In the mixed counties, there is some variation in the proportion of districts gaining, but in each case the dominant district does not gain. The dominant district gains in all of the rural counties, although the proportion of districts gaining ranges from 33 to 67 percent. Thus, this county equalization feature provides tax relief to dominant districts in urban and rural counties and not in mixed counties.

Table 8. Redistribution effects of Plan 2's county equalization provision (1967-68 Iowa data)[a]

County	Equalization No. of districts Showing gain	Percent	Effects on Dominant district Gain	Loss
A (Urban)	5	56	X	
B	1	25		X
C	6	55		X
Total mixed	7	44		
D	2	33	X	
E	2	50	X	
F	2	67	X	
Total rural	6	46		
Grand total	18	48		

[a]Calculated from 76.

Effect of Plans Distribution on Equity

Equity has been listed as one of the distribution effects of state aid grants. In this context, equity refers to individual educational needs relative to the distribution of educational services and to the distribution of the burden of paying for them.

The equity effect in relation to the burden, is investigated in this study. Burden is herein defined as the local tax askings per student. This figure is ratio of the difference between total district expenditures and state aid, and the number of students in the district.

This particular investigation sought determination of the extent to which inequality remained after the institution of a given plan. For this purpose, inequality was defined, operationally, as differences in county burdens.

For each plan, a one-way analysis of variance of tax-askings per student was performed across counties using district parameters. Hence, the mean burden per county was compared for each plan. The means and standard deviations are in Table 9. The null hypothesis tested was that no significant differences existed in district burden per county. The F values are presented in Table 10. This procedure differed from that employed in the analyses of property tax relief and finance (which follows). In these cases, the primary means of determining plan effects was to evaluate district parameters across plans. Contrariwise, the equity effects were determined by comparing plan parameters for each district across counties. For the entire cross-section of urban, rural and mixed counties,

Table 9. Burden per student (1967-68 Iowa data)[a]

		Plan 1	Plan 2	Plan 3	Plan 4
6 county	$\bar{X}$[b]	579.417	509.532	608.670	609.172
	co.v[c]	0.295	0.128	0.332	0.331
A	$\bar{X}$	530.242	499.962	519.869	533.408
	co.v	0.138	0.230	0.148	0.178
B	$\bar{X}$	440.814	436.612	459.526	445.588
	co.v	0.201	0.243	0.235	0.243
C	$\bar{X}$	510.049	468.947	502.907	509.290
	co.v	0.164	0.270	0.193	0.191
D	$\bar{X}$	918.562	620.163	1004.613	1005.715
	co.v	0.786	0.258	0.716	0.725
E	$\bar{X}$	567.555	547.828	629.050	610.775
	co.v	0.342	0.384	0.320	0.368
F	$\bar{X}$	509.280	483.677	536.053	550.253
	co.v	0.195	0.209	0.278	0.220

[a]Calculated from 70, 75, 76.

[b]$\bar{X}$ = mean.

[c]co.v = coefficient of variation.

Table 10. Effect of plans (columns) on equity as measured by F values for all the districts in the six county area (1967-68 Iowa data)

Plan number	Plan effect
1	1.490
2	1.361
3	2.733*
4	2.612*

*Significant at five percent level.

the null hypothesis was accepted given the implementation of Plans 1 and 2 and rejected given the implementation of Plans 3 and 4. It should be remembered that under Plan 2, income was included in the measurement of district wealth per student relative to the state. Also, the distribution of wealth in a district relative to neighboring districts within its county was accounted for in Plan 2. Plan 1 was based on a fixed amount per student. Plan 3 incorporated an ability to pay measurement based on property values. Plan 4 was a mixture of a flat rate per student and a reward for effort.

The similarity of the equity implications of Plan 2, the most sophisticated equalization formula, and of Plan 1, the nonequalization formula, is striking. From this similarity, it can be concluded that if the burden segment of equity is the distribution effect to be heavily weighted in policy formulation, the formula chosen must be sophisticated enough to explicitly account for the realities of the economic situation. If this is not feasible a simple cost reimbursement based on the distribution of students is the best alternative.

Effect of the Form of Finance on Distribution Plans

As mentioned in the theoretical discussion, the economic structure of the state was thought to be a very important determinant of the impact felt in school districts as a result of choosing the form of state aid distribution. The state tax system was one element of the set composing the economic structure.

In an attempt to ascertain the effect of the interaction between the type of state grant, and the state tax system on the distribution of state aid, the following analysis was performed. A fixed amount of total state aid was determined. This amount was to be fully generated by both the income tax and the sales tax. In each district the net gain (loss) per student was calculated for each plan. This was the difference between the aid the district would receive under the plan and the tax the district pays to the state. This net income was calculated for the income and the sales tax and divided by average daily membership.[1] The estimate of income tax collected from each district was determined by calculating the percent of adjusted gross income generated in each district in the six county "state."[2] This percent was multiplied by fixed amount of total state aid previously specified. As state income tax is proportional to a certain extent, there should be little distortion in the above transformation. District sales tax paid was also approximated because only county sales tax data were available.[3] Each district's average daily membership was expressed as a percent of county average daily membership. This percent was multiplied times the county sales tax collections. This amount was each district's imputed sales tax contribution. District contribution summed over all the districts yielded total "state" sales tax collections. (Summing county sales tax data already provided yielded the same total.)

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[1]The source of this is 1966-67 Iowa data provided by (70).

[2]The source of this 1967-68 Iowa data is (76).

[3]The source is (75).

The district percent of the state total was determined. This percent was multiplied times the predetermined amount of state aid to yield the state sales tax contributions.

Table 11 summarizes the mean gain per student under the four plans for each of the two methods of finance. For the six counties taken together, there is a net gain under Plan 1 regardless of the financing. Plan 2 shows a small average net gain only when financed by an income tax. Scanning the individual county averages, it is observed that county A, the most urban of all the counties receives a net gain under all income tax financed plans and an average net loss for all sales tax financed plans. The results are less clear for the mixed and rural counties. Counties B and C (mixed), record net gains when Plan 1 and Plan 4 are financed both by the income tax and by the sales tax. Plan 2 causes a net loss under both financing arrangements. Again for rural, counties, Plan 1 yields an average net gain under both forms of financing. Plan 3 causes a net loss in the same two rural counties under both forms of financing. In order to determine the statistical significance of the financing on the plans a two way analysis of variances were calculated. See Table 12.

Based on six county averages, neither the type of financing nor the county has any effect on the loss or gain per student. However, looking at each county and classifying it according to the urban, mixed and rural categories yields heterogenity not apparent in the averages. In A, the urban county, the finance effect is significant for all of the plans. In mixed counties, there is no clear trend. In county B none of the plans are effected by the form of finance. This is in almost direct contrast

Table 11. Effect of form of finance on distribution of plans based on per student net gain or loss (1967-68 Iowa data)[a]

		Income tax				Sales tax			
		Plan 1	Plan 2	Plan 3	Plan 4	Plan 1	Plan 2	Plan 3	Plan 4
6 county	$\bar{X}$[b]	28.529	0.569	- 2.292	- 1.226	20.772	- 7.186	- 4.413	- 8.983
Av.	co.v[c]	0.305	26.801	-15.901	-27.540	1.161	- 2.931	- 7.176	- 2.671
A	$\bar{X}$	27.318	22.948	37.691	24.149	-14.982	-19.350	- 4.608	-18.150
	co.v	0.756	4.582	1.480	1.221	4.915	- 5.178	-10.962	- 1.580
B	$\bar{X}$	22.048	-15.072	3.334	17.272	16.120	-20.998	- 2.594	11.346
	co.v	1.127	- 2.288	8.677	1.581	0.067	- 0.656	- 7.712	1.794
C	$\bar{X}$	21.533	-14.369	19.260	22.292	1.867	-35.034	9.011	2.627
	co.v	0.914	- 1.769	2.290	1.035	0.963	- 0.367	3.057	6.453
D	$\bar{X}$	21.528	- 0.760	-64.523	-65.623	33.555	11.265	-52.500	-53.602
	co.v	1.749	-131.707	0.609	- 0.659	0.064	7.073	- 0.110	- 0.339
E	$\bar{X}$	38.538	-1.465	-22.952	- 4.682	42.190	2.192	-19.300	- 1.028
	co.v	0.622	-39.014	- 1.255	- 7.562	0.013	15.848	0.907	-34.461
F	$\bar{X}$	40.207	13.133	13.437	- 0.763	45.883	18.810	43.513	4.910
	co.v	0.731	3.614	7.114	-34.129	0.048	1.110	1.366	4.118

[a]Calculated from 70, 75, 76.

[b]$\bar{X}$ = mean.

[c]co.v = coefficient of variation.

Table 12. Effect of form of finance (columns) and counties or districts (rows) in the distribution characteristics of plans as measured by F values (1967-68 Iowa data)

Plan number	Geographic area	Finance effect	County or district effect
1	6 county	0.885	2.223
	County A	36.443*	0.950
	County B	0.300	1.113
	County C	11.031*	1.023
	County D	0.571	0.871
	County E	0.091	0.964
	County F	0.111	0.995
2	6 county	0.884	2.315
	County A	36.445*	94.491*
	County B	0.300	3.713
	County C	11.031*	3.692*
	County D	0.570	20.524*
	County E	0.091	14.291*
	County F	0.111	5.168
3	6 county	0.046	6.863*
	County A	36.443*	24.626*
	County B	0.300	3.225
	County C	1.103	4.160*
	County D	0.570	1.075
	County E	0.091	2.885
	County F	1.437	12.419
4	6 county	0.885	7.409
	County A	36.443*	6.656*
	County B	0.300	2.963
	County C	11.027*	3.242*
	County D	0.570	1.894
	County E	0.091	7.750*
	County F	0.111	1.496

*Significant at five percent level.

to County C where all but Plan 3 are influenced by the type of tax. In the rural counties none of the losses or gains under the four plans are affected by the tax system. The district effect is negligible in rural, urban, and mixed. Thus, in an urban setting, it is important to consider the source of finance when evaluating alternative forms of state grant distribution. Sales tax financed plans causes net losses. The tax structure is not as important a consideration for rural units.

Summary and Conclusions on the Distribution Effects of State Aid

There are regional patterns to the array of state grants presently in use. However, these patterns are changing. Over the five year period from 1957 to 1962, the number of nonequalizing plans in effect throughout the United States increased, while the number of plans designed to equalize decreased. Of the nonequalizing grants, the importance of variable types had increased. Among the equalizing plans, the percent equalizing type showed the largest increase in usage. The variable nonequalizing plan showed the largest gain in usage among the nonequalizing types. As of 1962-63, the fixed unit nonequalizing plans accounted for 46 percent of the total plans in operation and was therefore, the most prevalent state general resource distribution plan in the United States.

Of the plans analyzed, Plan 1 is a representative of the fixed, nonequalizing type, while Plans 2 and 3 are representative of the percent equalizing type. Plan 4 is a combination of two types, fixed nonequalizing and variable equalizing. The purpose of the empirical analysis was to test the null hypotheses that: (1) there are no differential distribution effects (property tax relief and equity) to be found among the alternative state aid allocation plans given varying economic conditions, and (2) the allocative effects of the plans are not affected by the form of finance used by the state. Six Iowa counties made up the test sample. They were classified as urban, mixed and rural counties.

With respect to the tax relief redistribution effect, Plan 2 which included revenue from income tax sources allowed the lowest millage level both within and over counties. With the exception of Plan 1, none of the other plans had a statistically significant effect on millage level either across counties or within districts. Within one rural county, Plan 1 produced significantly lower millage levels than Plans 3 or 4. Both over counties and within counties, the county and district respectively did have a significant effect on millage.

On the basis of an arithmetic comparison, Plan 3 lowered millages in large urban districts, but not in moderately large, mixed or rural districts. Plan 4 was not helpful in relatively large districts. The lowest millage level in rural districts was obtained under Plan 1. The District equalization feature of Plan 2 did provide a measure of tax relief to large or dominant districts in urban and rural counties, but not mixed counties. Little generality can be attached to the findings of the arithmetic comparisons. As mentioned, the differences were not large enough for statistical significance. However, small differences in millages have a large impact on tax bills.

The district equalization feature of Plan 2 did provide a measure of tax relief to large or dominant districts in urban and rural counties, but not in mixed counties. Thus, the effect of plan type used on property tax rates is negligible. It becomes significant only when a comprehensive measure of wealth (i.e., including income) is included in the formula.

With respect to the equity effects, it is concluded that unless a determined effort is put forth to include the realities of the state's economic-demographic configuration as in Plan 2, a simple fixed unit formula such as Plan 1 allows for the most equalized distribution of the burden.

Perhaps the most significant finding of the investigation is the following: given the form of state aid distribution, the type of finance used to implement the plan is crucial to urban areas. In urban areas, the form of finance made significant differences in loss or gain per student in all of the plans. Thus, the tax structure is not as crucial to the plan effect in the mixed and rural counties.

The generality of the findings is limited by the nature of the sample (i.e., the degree of difference between the counties labeled rural through urban), the relatively low state share of total local educational expenditures in Iowa (approximately 20 percent) and the absence of income from the measure of wealth used in Plans 3 and 4.

However, decision makers in all states should be aware that plans for the allocation of general aid may not yield the results which are dictated by practice theory. This divergence arises from variations in regional economic structure. The consequences of any plan should be carefully investigated.

CHAPTER IV. THE OUTPUT EFFECTS STATE SCHOOL AID PLANS

Introduction

The resource distribution effects of state aid plans were discussed in Chapter III. These effects were analyzed in a framework in which schools are viewed in a manner Benson called "standardized-opportunity-systems" (8, p. 1). Benson, in referring to the move toward a systems analysis approach to educational finance, states that under this framework schools are viewed as "differentiated-output-accountability systems" (8 , p. 1). Under the first approach the success of the school district (or state educational system) is judged on the basis of the inputs (educational services) it makes available to its students. Under the second approach, the educational system is held responsible for the achievement of its students.

In order "to establish a cash nexus between the interests of the state government in seeing school districts move toward higher levels of performance and the earning capacity of local districts," Benson suggests a system of state aid consisting of two parts: first, a basic program in which the state assists districts in bringing the achievement levels (measured in alternative ways) up to some minimum level, and second, a local improvement fund from which grants are distributed (inversely to the district's ability to pay) to stimulate further advances in achievement (8, p. 4). Following Benson's pattern of thought a question as to the extent of the output effects of the state aid programs currently in use arises. Research directed to this question is discussed in the following chapter.

The Statewide Primary-Secondary Education Model

Description of the model

In order to analyze the output effects of state aid to primary and secondary education, the following model was developed. In the framework of the model each school district in the state is considered to be a process in the linear programming sense. That is, each district starts with students of a given type. The students are run through the process by means of the application of given amounts of resources. These resources include the services of teachers, buildings, teaching materials, administrative and auxiliary personnel. The proportions of the resources used varies between processes (school districts). The optimal use of resources within a process is a micro-economic problem and will not be discussed in Chapter IV. For present purposes, the fact that the application of resources within a process can be represented by the dollar cost per student is most relevant. Hence, each district incurs a given cost to produce an educated student. The level of education, or achievement, is assumed to be measured by the standardized achievement tests currently in use. The amount of funds each district has to spend in a given year is the sum of what it raises locally plus what it receives outside aid. The output effects of outside aid (state aid in this case) can be analyzed if it is assumed: (1) that the goal of the people of the state is to have their students attain the highest possible level of achievement, and (2) that (as Benson suggests) state aid is to be used to further this end.

Of course, the amount of funds to obtain the resources for education is limited. Primary-secondary education as a public good must compete for funds with other public goods such as health services, recreation facilities, highways, police and fire protection as well as with the citizens' desires for private goods.

Thus, the problem resolves itself into an economic problem of allocating scarce resources among diverse and competing ends (6, pp. 70-129). The problem can be restated as one of maximizing the achievement level of the state's students subject: (1) to local budget constraints, and (2) to a state spending constraint. Now, each district is viewed as a process which is competing for state funds. Additional funds to provide the resources for the districts could come from local sources, but it is generally considered that the ceiling has been reached on taxing of local wealth. Hence, additional aid must come from the state. In this framework, state funds will flow to the districts based upon where they can do the most good with respect to meeting the objective. The iterative process continues until the gain per dollar is equalized in all districts.

The basic model is set down in its entirely and then is explained. The variables are defined as:

1) X_{ijklm} = the number of students of quality class i, attaining achievement test score level j, in the lth school of the kth district of the mth county where:

$i = 1,2,3$
$j = 1,2,3$
$k = 1,2,3.....K$; K is the number of districts in the mth county
$l = 1,2,3.....L$; L is the number of schools in the kth district
$m = 1,2,3.....M$; M is the number of counties in the state.

2) $X_{ijkm} = \sum_{l} X_{ijklm}$

3) SS_{km} = state aid to district k in county m

4) $SS_{m} = \sum_{k} SS_{km}$ = aid to the county

5) $SS = \sum_{m} SS_{m}$ = total state aid

6) LS_{km} = local spending of the kth district of the mth county

7) W_{ijkm} = the achievement test score of students in the ith quality level of the kth school district in the mth county

8) N_{im} = the number of students of the ith quality level in the mth county.

The objective may be stated as:

$$\text{Max } Z = \sum_{i}\sum_{j}\sum_{k}\sum_{m} W_{ijkm}X_{ijkm};$$

subject to the following constraints:

1) score constraints, for each m:

$$\sum_{j}^{3}\sum_{k}^{K}(W_{1jkm}X_{1jkm}) / \sum_{j}^{3}\sum_{k}^{K}(X_{1jkm}) \geqq a;$$

where $a = \sum_{j}^{3}\sum_{k}^{K}\sum_{m}^{M} W_{1jkm}X_{1jkm} / \sum_{j}^{3}\sum_{k}^{K}\sum_{m}^{M} X_{1jkm};$

2) student constraints, for each m:

$$\sum_{j}^{3}\sum_{k}^{K} X_{1jkm} = N_{1m},$$

$$\sum_{j}^{3}\sum_{k}^{K} X_{2jkm} = N_{2m},$$

$$\sum_{j}^{3}\sum_{k}^{K} X_{3jkm} = N_{3m};$$

3) budget constraints for each district k:

$$\sum_{i}^{3}\sum_{j}^{3} a_{ijkm}X_{ijkm} - SS_{km} = LS_{km} \text{ for each county m:}$$

$$\sum_{k}^{K} SS_{km} - SS_{m} = 0, \text{ and}$$

4) budget constraints for the state:

$$\sum_{m}^{M} SS_{m} - SS = 0 \ .$$

The number of constraints are as follows:

2M-score constraints
3M-student constraints
KM-district budget constraints
KM-district aid constraints
M-county aid constraints
1-state aid constraint

6M + 2KM + 1 constraints is the total number.

Variable 1 represents the output of the ith school in the kth district of the mth county. The subscript i, representing quality class, is necessary to account for the different types of students to be processed in each school. The subscript j represents the level of achievement.

The choice of three levels is arbitrary. The score on an I Q test or some other standard can be taken as a measure of the quality or status of the student entering the process. Therefore, schools start with different qualities of students who then emerge with various levels of achievement. One presumes the school to be responsible for the major portions of quantitative skill levels and verbal skill levels measured by the achievement tests.

In terms of the model under discussion, there are nine combinations of students that can emerge from the process. That is to say, the school can start with a student of input class 1 and bring him to achievement levels 1, 2, or 3. Variable 2 represents the number of students summed over all the schools in the district. This is the basic unit used in the empirical demonstration of the model because data are generally available by district rather than by school. Variable 3 represents state aid to the district. Aid to the county is determined by summing over the district aid (variable 3). This variable is of primary interest because the model was constructed on an intermediate (regional) rather than a micro-educational or district level. The students are to be educated by alternative processes that are available in the county. Variable 5 is the amount of state aid determined by the legislature. Variable 7 represents the pay-off or final level of achievement of given class of students. Variable 8 stands for the number of students of a given input class in the county. A county can have all three types or only one type of student input.

The objective function to be maximized is simply the weighted total of the student achievement test scores of all three types of students in the entire state. Division by the number of students in the state yields the average score per student.

The score constraints indicate that the students of type one and two in each county should achieve average scores at least equal to "a" and "b" respectively. If desired, "a" and "b" can be set equal to the average achievement scores earned by the respective groups on a statewide or county-wide basis.

There is no constraint on the scores to be earned by type 3 students. Alternative specifications of this constraint are possible. The standards (i.e., "a" and "b") could be calculated as percentages of the score earned by the students of beginning quality type 3.

Also, these constraints could be specified on a less restrictive statewide basis. That is, the score earned by students of input type 1 and type 2 over the entire state should reach a certain level. The students in one county might be above the requirement and the students in another county could be below the norm. No requirement was put on the achievement level of high quality students. After allocating state aid to bring up the first two groups to the required level, the program allocates the remaining funds to the activities yielding the highest level of points per dollar of expenditure.

It was assumed that the marginal productivity of a dollar invested in high quality students would be higher than for any other group and hence the high quality student would be brought up to the highest feasible level. There is a trade-off on the norms to be reached by the various types of students. By fixing "a" and "b" at progressively higher levels the range for increasing the achievement of higher quality students decreases. In more practical terms, given a level of expenditures, the more spent on remedial training, the less that is available for expenditure on laboratories and college preparatory courses.

The student constraint indicates that in each county all of the students, regardless of their initial input quality must be educated. If it were not for this constraint, the program would search out the activities yielding the maximum score per dollar and all of the students would not enter the solution vector. This would be unrealistic. In the model, all of the students are educated somewhere in the county. This does not necessarily mean that the students are transferred between school districts in the county, but that the procedures used in a particular district can be duplicated by other school districts in the county. In other words, the conditions between counties should be homogeneous to the extent that, given enough funds, any district in the county could attract qualified personnel or construct the facilities necessary to educate the students of each type in the optimal fashion. This presupposes the presence of rational decision makers and the absence of barriers to the employment of the necessary methods. Also, it is recognized that no technique can be copied exactly due to differences in personnel, facilities and the nature of the students. An alternative would have been to build the consolidation of schools and the transportation of students into the model.

Local spending within each district is the difference between the two indices: the amount spent to educate each type student (X_{ijkm}) multiplied by the number of each type of student produced, and (2) state aid. The difference between state aid over all districts within the county and state aid to the county is zero as is the difference between total state aid and state aid summed over the counties. The last three constraints are accounting identities.

Uses of the model

The model, as explained above, can be used in several ways to ascertain the output effects of state grant distribution plans. Given an initial set of constraints, the minimum amount of state aid necessary to meet these constraints can be determined by backward recursion. An optimal distribution of aid to each county is implied. Alternate methods of distributing state aid can be compared to the optimal. The score constraints can be increased and the optimal solution again determined and compared with what would exist under alternative distribution plans. This procedure is necessary to find out whether the output implications change as the score constraint changes.

Instead of finding the minimum amount of state aid necessary to meet a given set of constraints and relating its distribution to various plans, an alternative approach is available. It can be assumed that the state is willing to fully fund alternative plans for state aid distribution; these totals can be fed into the state aid activity on an unrestricted basis. It becomes a question of whether there are efficient distributions of these particular amounts. In other words, how close is the distribution of aid under a particular plan to the optimal distribution of the amount engendered by full funding of that plan? This is the dual of the method described above and would yield comparable information.

The model can be used to analyze the output effect of the migration from rural to urban. This can be accomplished by moving a proportion of the students from rural to urban areas and observing the change in the minimum amount of state aid necessary to meet a given score constraint set.

Thus, the decision makers in the state can be given an indication as to whether this migration will lead to increasing or decreasing county demands for aid in order that goals may be achieved.[1]

Implementation of the Model

Determination of the sample

It was decided to draw a larger example than was analyzed in the analysis reported in Chapter III. In general, a sample may be drawn on a restricted or unrestricted basis. Under the latter method, each sample member is selected from the population at large in such a way that each member of the population has an equal chance of being selected in the sample. Unrestricted sampling was not used, because of the danger that the sample would not be representative of the population. There is always the possibility that a large proportion of the sample drawn can have a particular characteristic. Thus, it is a possibility that other segments of the population with other characteristics could be under represented. When there is considerable heterogenity in the population it is considered good practice to divide the population into strata from which specified proportions of the sample are drawn. This insures adequate representation from all the relevant segments. The number of sample members can be taken in proportion to the differing degrees of heterogenity. The idea is to draw more heavily from the strata displaying the higher variability. This reduces the sampling error for a sample of a given size (25, pp. 245-251).

[1]The model, in the static form presented, does not account for new consolidations of school districts, inflation nor advances in educational technology that have differential effects in urban versus rural areas.

It is recognized that two of the more important influences on the education process is the type of socio-economic area in which the district is operating and the size of the district.[1] The school districts used to generate empirical content for the model come from the State of Iowa. The data are for the years 1966-67 (70) and 1967-68 (60). For the first application of the model it was desirable that a representative sample be chosen.[2] Hence the school districts of the counties in Iowa were stratified on the basis of area, according to three U.S. Census area type delineations. These are: urban, as represented by the Standard Metropolitan Statistical Region (SMSA), rural and mixed; the latter designated by the Census as an "Urban Place". An SMSA county is defined as an area having unincorporated and incorporated places of greater than or equal to 2500 inhabitants. Rural areas are defined as those where people live on places of greater than or equal to 10 acres from which sales of farm products amounted to greater than or equal to $50.00 in 1959 plus the non-residents of the adjoining area (80, pp. vii-x). On a strict proportionate basis the districts in the SMSA account for 7 percent of the state's students while rural

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[1] See the following references: 26, pp. 7-24; 29, pp. 26, 33, 43; 30; 42, p. 13; 79, p. 5.

[2] When emphasizing the resource distribution effects, the economic base of the county is of importance. In agricultural economics, the geographic location is a determinant of the economic base. Also, the spatial pattern of large and small school districts is another determinant the relative ability to pay for education. Therefore, in the analysis presented in the previous chapter, the counties of the state were stratified on the basis of location and the size distribution pattern of school districts to choose the sample.

districts account for over 86 percent. However, there is more variation with respect to size of district within the SMSA classification than in the rural classification. This fact made it more desirable to get additional observations from the SMSA regions than would be drawn from a strictly proportionate sampling. In order to accomplish this, the proportions were weighted by the variances of school district size in each area. On this basis it was determined that the sample be drawn such that 21 percent come from SMSA's, 77 percent from rural and 1 percent from urban places. A sample of approximately 20 percent of the 99 counties in Iowa was decided upon. On this basis four SMSA, one urban place and 16 rural counties were drawn at random. In order to test for differential regional effects, it was necessary that economic regions be used. Two of Iowa's more well known regions, one in the northern (NIAD) and the other in the southern section of the state (TENCO), were chosen.

Determination of the coefficients

W, the output measure used in the model, was the average composite score made by seniors on the Iowa Test of Educational Development (ITED) for each district. Determination of the relative merit of one type of achievement test versus another is outside the scope of this study. The ITED Battery is generally considered to be a reliable test of general educational achievement. Differences from school to school in the content and quality of instruction and in other academic factors do show up in the ITED scores (33, p. 63; 48, p. 21). Of course, differences in environment also show up here, but the fact that averages rather than individual scores for the districts are being used acts as a control factor (33, p. 63). In other words, a measure of the results of each process (district educational systems) is required and achievement test scores such as ITED provide a good approximation.

The ITED composite score is a standardized summary of the scores earned on eight individual tests. These eight tests cover understanding of social concepts, background in the natural sciences, ability to do quantitative thinking, ability to interpret reading materials in social studies, natural sciences and literature and general vocabulary (33, p. 6).

The actual measurement of the input classification as indicated by subscript "i" created difficulty. Ideally, average I Q for the students in each district would have been desirable. Such data were not available. Based on reported findings (33, p. 30), Thomas (77) used the ITED vocabulary test score as a proxy for intelligence and it was so used in this study. Frequency distributions of the average district 1967-68 ITED composite and vocabulary scores were prepared. Each distribution was arbitrarily divided into three classes, such that each class accounted for one third of the observations.[1] This determined the score designations of low, medium and high. The average score on the composite test made by seniors of a given district was classified as one of the three levels based on their average vocabulary score. Hence for the kth district in the mth county W_{ij} and X_{ij} were determined. Next, a determination of the number of each type of student in the county, N_{im} was made. The specification of the score constraints was

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[1]The dividing lines were 14.0-18.69 (low), 18.70-19.79 (medium) and 19.80-23.30 (high) for the vocabulary scores. For the composite scores the benchmarks were 15.90-20.99 (low), 20.10-21.49 (medium) and 21.50-25.59 (high).

straight forward, though arbitrary. There are three possible results for the ith input group. They can be processed such that the average score per student could be at the low, medium or high achievement levels. Thus, the N_{im} students in the county can be educated under processes whose results are X_{11}, X_{12}, or X_{13}. The constraint specification states that the average score of the group must be at a certain level, either a or b in the model. Students can be educated under one process if the average score is greater than or equal to the prescribed level or by a combination of processes. Thus:

$$W_{11}X_{11} + W_{12}X_{12} + W_{13} \geqq aN_{1m}; \text{ then, dividing by } N_{1m} \text{ yields:}$$

$$W_{11}X_{11}/N_{1m} + W_{12}X_{12}/N_{1m} + W_{13}X_{13}/N_{1m} \geqq a \, .$$

The specification of values for "a" and "b" has been discussed. Each X coefficient is weighted by the quotient of the score of the students completing the course of study and the number of students of the ith type in the county. There are two such constraints for each county. The process of optimization determines the level of each process used. It distributes the students among the process until it finds the most efficient way of satisfying the score constraints.

The output measure described above, represented the ITED composite score earned by seniors. This score measures the outcome of a process that takes twelve years. Therefore, the cost per student per year in the kth district (calculated by dividing current yearly expenditures by number of students) is not the full cost of generating students earning a given score.

A measure of the twelve year cost or expenditure per senior was determined. The general fund expenditure recorded each year by the school district represents the cost of operating all the grades in the school system. If this expenditure were divided by the number of seniors, the result would represent the cost per senior only if the number of students in each year of school was identical. An examination of data revealed that this was not the case. Therefore, cost per graduating student was estimated by: (1) dividing total general fund expenditures by total number of students; this yielded cost per student per grade per year, and (2) multiplying calculation (1) by 12; doing this yielded the twelve year cost for a senior student expressed in current prices.

Given this measure of cost per senior student, the use of general fund expenditures, as published by each district, in the local budget constraint equations, would have been inconsistent. This was because the graduating seniors of a given year were not actually educated at this cost. Costs have been generally increasing, hence the actual cost of the first eleven years of the current year's budget would imply more local financial resources were available per senior student than actually was the case. Therefore, the local budget constraint was calculated by multiplying the expenditures per senior estimated above by the number of seniors.[1] This calculation

[1]This cost coefficient also does not account for upgrading in the educational process. The same amount spent in future years may obtain higher quality inputs and hence better educated students. However, since in a given year the cost per senior per year represents a kind of twelve year moving average, the temporal factors discussed balance out.

In order to illustrate the procedure the following example taken from an urban district is offered:

General Fund Expenditures....$1,236,458
Number of Students (Average Daily Membership)....2,696.5
Cost per student per class...$485.56; Cost per student per 12 years...$5503; Number of seniors taking ITED...156.
Local Budget.............156 x $5503 = $858,466.

assumed costs per class year were equal. This figure was generally smaller than the actual yearly expenditures per district due to the fact that the dropout rate gradually reduces the size of the senior class. The divergence between actual expenditures and budget constraint used in the model would vary according to the extent of inflation, the differences in cost per class year and the dropout rate in each district. At the time the calculations were performed, the 1967-68 expenditures were not yet compiled by the State of Iowa, so the 1966-67 financial data were used.

Results

For each geographical area, three solutions to the model were determined. There was a solution for each of the three sets of score constraints. See Table 13. The constraints apply to type one and type two students. Under the first or original set of constraints, it was required that the type one and type two students in each county achieve minimum scores of 19 and 20 respectively. These levels approximate the state averages for 1967-68 (60). Under the second or set of average constraints, it was quired that the students in each district score at least as well as the county average for their group. Under the third set of maximum constraints, it was specified that the students from each district within a county attain a score equal to highest score achieved by students of the same classification elsewhere in the county.

For the basic twenty-one county sample, $8,604,195 is the minimum amount necessary to generate a solution (given the original constraints). The last dollar of state aid allocated to the counties yields 0.0737 points.

Table 13. Results of model run for three sets of constraints and nine areas

Constraint	Area	Total amount of aid ($)	Reduced cost ($)	Objective function	Number of students	Average score
Original	All 21	8604194	0.07137	278540	13407	20.775
	Urban	8114277	0.05429	195802	9399	20.832
	Rural	489916	0.07137	82738	4008	20.643
	21+T+N	9311900	0.07137	330434	15929	20.746
	21+T	9128210	0.07137	309087	14927	20.707
	21+N	8779892	0.07137	299921	14408	20.816
	T[a]	523945	0.03464	30513	1520	20.075
	N[b]	183699	0.04245	21381	1001	21.359
	T+N	707644	0.04245	51894	2521	20.584
Average	All 21	15249273	0.07137	279676	13407	20.860
	Urban	12441966	0.05429	196191	9399	20.873
	Rural	2807581	0.07137	83485	4008	20.823
	21+T+N	15853853	0.07137	331563	15928	20.814
	21+T	15375704	0.07137	310169	14927	20.779
	21+N	15727800	0.07137	301076	14408	20.896
	T[a]	126431	0.03464	39487	1520	20.006
	N[a]	478149	0.04245	21400	1001	21.378
	T+N	604580	0.04245	51887	2521	20.582
Maximum	All 21	39999158	0.05429	282519	13407	21.072
	Urban	31296012	0.05429	197688	9399	21.033
	Rural	8702929	0.04136	84831	4008	21.165
	21+T+N	43358400	0.05429	334796	15929	21.018
	21+T	42221200	0.05429	313359	14927	20.993
	T[a]	2222332	0.01488	30838	1520	20.288
	N[b]	1137146	0.00344	21401	1001	21.380
	T+N	3359517	0.01488	52277	254	20.737

[a]T = TENÇO.

[b]N = NIAD.

This is the entry in the reduced cost column. Conversely, removal of a dollar from the total amount of state aid would reduce the value of the objective function by 0.07. For the 13,407 students in the 21 counties the average score per student is 20.775. To fulfill the conditions specified in the set of average constraints, the amount of state aid necessary nearly doubles ($15,249,273). The reduced cost or marginal product is the same. The average score per student increases to 20.860. It is required to more than double the amount of state aid to $39,999,153 in order to meet the maximum constraints. The average increases to 21.072.

The reduced cost is affected by the region. In urban counties it is slightly lower than in rural (0.05 versus 0.07). The functional economic areas TENCO and NIAD have differing effects on marginal product. When the state is considered to consist of only NIAD and TENCO, the reduced cost takes on the value of NIAD alone (0.04). When NIAD and TENCO are attached to the other 21 counties either separately or together, the reduced costs remain the same (0.07). Because the score constraints of the model are constructed such that the county was the primary unit, the amounts of state aid going to each county to meet the constraints remains the same given all the combinations of runs. The amount a county in the original 21 county group receives does not change whether TENCO or NIAD are added to the group or whether just the rural or urban group is run alone.

A substantial increase in an expenditure yields a relatively small gain in output. This indicates that increasing the quality of education received by the students in the sample is expensive. The elasticity of score with respect to state aid was defined as the percent change in score divided by the percent change in state aid. It was computed in the following manner. The average score and the amount of state aid necessary under the original bounds were used as base quantities. The score and state aid associated with the average and maximum constraints were divided by the base quantities. This yielded adjusted scores and adjusted state aid quantities. Thus, the first interval elasticity approximation was the quotient of adjusted score achieved under the average constraints. The second interval approximation was generated by combining the adjusted state aid amounts for the maximum constraints in the same way.

The first coefficient of elasticity is 0.6539 for urban counties and 0.1760 for rural counties. The second coefficient is 0.2618 and 0.0577 respectively. Thus, the impact of state spending in urban areas is initially greater than in rural areas. As larger amounts of state aid are received, the impact of the funds in both areas diminishes, but the differential is maintained. The same trend occurs in the two functional economic regions although the elasticities computed for TENCO are larger than NIAD's.

In interpreting the absolute amounts of state expenditures it should be recalled that in order to increase students' achievement to the desired levels the model was constructed under the assumption that the educational techniques available within a given county may be implemented by a given district in that county. Hence the start up costs would be funded by the state aid to the county. The analysis was static in that the implementation of techniques employed outside of the county or new techniques not yet employed anywhere in the state were not considered. The reason for the exclusion was that the analysis was directed towards the question, given the present status of the educational system within the counties of the state, what redistribution of resources is required to maximize the level of educational output? It was possible that the implementation of new educational processes could increase the potential of the system.[1]

Analysis of the output effects of alternate distribution plans

The optimal distribution of aid to the twenty one county group for each of the three sets of score constraints is presented in Table 14. To meet the original constraints, state aid, as already mentioned, of $8,604,144 is the minimum amount necessary. In order that this amount be sufficient, it must be distributed in the manner shown. County 1 is to receive the most aid and several counties (numbers 5, 9, 11, 12, 13, 20, 21) are not to receive any aid. The amounts under the average and maximum constraints are interpreted in the same way. The purpose of the analysis is to compare the optimum distributions with the alternative plans for distributing state aid.

The optimal distribution of aid to the twenty-one county group for each of the three sets of score constraints was compared to the distribution that would result from the allocation of an equal amount under alternate plans. Correlations and arithmetic deviation techniques were used.

[1]Staff and student inputs of a given class were assumed homogenous within a county. Hence the assumption of similar cost and output coefficients when a process from a neighboring district is instituted in a given district is not unreasonable. However, the use of coefficients from processes alien to the county would be suspect.

The latter were subsequently referred to as quantitative analysis, the former as qualitative analysis. The analysis of the separate regions specified earlier was terminated because the distribution results of the model were identical whether the model was run separately for each of these regions, for the two regions, for the twenty-one counties plus either region or for the twenty counties plus both regions. The more general urban-rural dichotomy was retained. Plan 0 refers to the optimal distribution. Plans 1 through Plan 4 were the same plans described in the previous chapter. Since the conversion in Iowa from Plan 1 to Plan 2 took place in the middle of the school year it was timely interest to analyze its output effects. Hence Plan 5 represents one half year's operation of Plan 1 and Plan 2.

Table 14. Distribution of the optimum amount of aid to each county ($)

County	Original constraints	Average constraints	Maximum constraints
1	7,324,928	0	9,787,492
2	0	7,721,200	12,198,700
3	0	4,269,916	7,764,760
4	789,349	425,152	1,349,672
5	0	25,698	195,599
6	116,905	461,897	1,247,680
7	30	1,199,038	1,667,013
8	317,847	0	427,829
9	0	0	279,297
10	173	157,096	1,042,748
11	0	33,474	0
12	0	0	386,583
13	0	0	0
14	29	35,973	255,806
15	64,930	17	407,543
16	0	0	133,898
17	4,152	207,058	1,083,183
18	850	850	850
19	0	452,327	1,257,794
20	0	259,577	263,002
21	0	0	249,701
Total	8,604,194	15,249,273	39,999,158

Qualitative analysis

Rank correlations between Plan 0 (optimum) and each individual plan (one through five) were calculated. The rank correlations measure the degree of association between each plan and the optimal plan. Rank correlation was used because the assumptions of this measure are considerably less stringent than those of product moment correlation. The most crucial assumption of the latter is the condition that the variables are bivariate-normally distributed. In rank correlation, nothing is assumed about the distribution of variables, yet it is still a relatively powerful test of association.[1] Also, the coefficient of concordance between Plans 0 through 5 and for Plans 1 through 5 were calculated. The measure is related to the average rank correlation coefficient.

The statistically significant[2] outcome of the procedures described in the previous paragraph is presented in Table 15. The term, r_{0i}, $i = 1,5$ represents the individual correlation coefficients. W_1 represents the coefficient of concordance among all six distributions and r_{ave-1} stands for the average rank correlation coefficient among the six. W_2 and r_{ave-2} represent the same measures for Plans 1 through 5.

Observing the first section (all 21 counties), it is noted that as the score constraints are increased from the original to maximum levels, the degree of association between the optimum (0) and each of the other plans increases. For example, r_{01} increases from 0.4416 (original constraints) to 0.6351 (average constraints) to 0.7256 (maximum constraints). Likewise W_1 and r_{ave-1} increase. W_2 and r_{ave-2} remain the same regardless

[1]See 64, pp. 202-213, 229-237; 88, pp. 467-472; 40, pp. 410-420.

[2]Significant at the 5 percent level.

of the constraint imposed. This is to be expected because, no matter what amount is distributed (i.e., the 8.5 million needed under original constraints through the 40 million needed to satisfy the maximum constraints) the proportionate share and hence the rank of each with respect to the amount of funds received remains the same for each plan. For the original constraints r_{01} is the lowest correlation and r_{03} is the highest. When the average constraints are imposed, r_{03} diminishes and r_{02} becomes the largest. When the maximum constraints are imposed, r_{01} is the highest while Plan 3 continues to have the lowest degree of association with the optimum allocation.

Unfortunately, the literature search did not yield a test to determine whether two rank correlations differ significantly from each other. If there were one it would turn on the form of the joint distribution of the difference between the hypothetical and actual correlation coefficients. Therefore, the most that can be concluded is that, given the statistical significance of the W_i's, there is not too much difference among the output implications of the plans for the 21 counties taken together. Possibly, the exception to this generalization occurs under the imposition of the average constraints. Under this condition, the relation between the Plan 0 and Plans 3 and 4 decreases more than does the relation between the optimal and the others. Also, as the output norms increase, all of the plans come closer to the optimum.

Observing, the second section of Table 15, rural counties, it is noted, given the average constraints, that all of the relationships are significant

Table 15. Correlations among optimal and alternative plans for the distribution of state aid

	r_{01}	r_{02}	r_{03}	r_{04}	r_{05}	W_1	r_{ave-1}	W_2	r_{ave-2}
I. All 21 counties Score constraint									
Original	0.4416	0.4545	0.4734	0.4546	0.4421	0.7639	0.7166	0.9134	0.8917
Average	0.6351	0.6377	0.4234	0.4949	0.6286	0.8161	0.7793	0.9134	0.8917
Maximum	0.7256	0.6685	0.6721	0.6952	0.7029	0.8573	0.8287	0.9134	0.8917
II. Rural counties Score constraint									
Original	0.5912	0.5632	0.5765	0.4397	0.5147	0.7343	0.6811	0.8228	0.7785
Average	0.0881	0.8346		0.5630	0.8096	0.7782	0.7338	0.8228	0.7785
Maximum	0.6581	0.5522	0.5184	0.5860	0.6184	0.7392	0.6870	0.8228	0.7785
III. Urban counties Score constraint									
Original	0.8500	0.9000	0.9000	0.9000	0.9000	0.9942	0.8930	0.9040	0.8848
Average						0.7631	0.7157	0.9040	0.8848
Maximum	0.9000					0.7888	0.7189	0.9040	0.8848

with the exception of the degree of association between Plan 3 and the optimum plan. In the rural section, it becomes more difficult to determine trends. As the score of the constraints becomes more binding, Plans 1, 2, 4, and 5 no longer show a progressively closer relationship between themselves and the optimal plan. The closest relationships occur under the average constraints and they diminish when the maximum bounds are imposed. There is a significantly close degree of relation among all five plans. However, when all counties are considered the coefficients decreased from the 0.9134 (all counties) to 0.8228 (rural counties). Perhaps the greater variation among the five plans is responsible for the weaker relation between each plan and the optimal plan in the rural category.

For urban counties, there is an equally close relationship between the optimal distribution and each of the other distributions under the original bounds. This extremely close degree of association ceases when the average and maximum constraints are imposed. There is considerable similarity among the distribution affects of Plans 1 through 5 (W_2=0.9040).

In summary, the qualitative statistical analysis, does not reveal large differences in output effects of the various plans. There can be various explanations of the above findings. The average rank correlation between the alternative plans is rather high. (It is lower for rural than for the total group. This might explain the more diverse effects in this group.) Thus, on a qualitative or ordinal basis there is not much difference among the plans. However, two plans can rank counties the same, but the dollar difference in aid tendered the counties can be substantial.

In other words each plan can rank counties A and B first and second among the 21 counties, but the dollar difference tendered A and B can be very small under one plan and large under another. A quantitative analysis follows. The other explanation of the relatively small difference in output effects among the plans is hinted at in Table 14 and in the elasticity discussion. Large amounts of money were needed to generate rather small changes in student achievement. In the optimal distribution, some counties receive large amounts and others receive nothing. Under all of the other plans studied each county receives something. Although the total is the same, the amount each county receives is diluted to the extent that the state aid does not represent a large proportion of the total expenditure for education in the county. Given the substantial amounts necessary to generate small changes in scores, it is conceivable that the absolute amounts received by the counties under alternative plans are not large enough to cause differential output effects.

Quantitative analysis

The quantitative differences between the optimal and the other plans were analyzed in two ways. The absolute value of the difference between the optimal amount per county and the amount to be distributed under a particular plan was calculated. This amount was summed over all the counties and divided by the number of counties (21). The absolute value was used to preserve individual county divergences and at the same time arrive at a total figure representing the overall deviation of any plan from the optimal. If absolute values were not used it was conceivable that the sum of the differences could be zero even though there was substantial departure from the optimal. The standard deviation of the differences was not used because it was felt that in the squaring process some perspective would be lost. The plan with the minimum sum of absolute deviations was considered to be the best. In the second analysis, only the additional amount per county needed to bring the aid it received under a given plan up to what it would receive the optimal distribution was considered. This amount was summed over the 21 counties.

The results of the first method of quantitative analysis are presented in Table 16. Table 16 is divided into three parts. Each part corresponds to a geographic unit. The units are the 21 county area, the urban counties, and the rural counties. For the 21 county unit, Plan 3 shows the smallest deviation per county, given the original set of constraints. The range of the differences from lowest to highest is $47,391. The pattern changes when the average and the maximum bounds are considered. Plan 2, which ranks next to the lowest under the original constraints, emerges as the closest to the optimum under both the average and maximum constraints. Plan 3 shows the largest deviation from the optimal distribution under both the maximum and average constraints. It deviated the least from the optimum under the original bounds. The difference between best and worst increases to $62,659 with the average constraints and increases to $156,028 given the maximum constraints. Under each constraint the dispersion is similar. (The range is about 10 percent of the average of the deviations.)

Table 16. Comparison of optimal and alternative distribution plans under varying score constraints

Constraint		Plan number	Average deviation ($)	Rank
		21 counties		
Original				
		1	482,295	5
		2	466,152	4
		3	434,904	1
		4	458,945	2
		5	466,101	3
	Range		47,391	
Average				
		1	894,739	4
		2	846,803	1
		3	909,462	5
		4	868,384	3
		5	877,739	2
	Range		62,659	
Maximum				
		1	1,183,005	4
		2	1,066,030	1
		3	1,222,088	5
		4	1,119,766	3
		5	1,110,545	2
	Range		156,028	
		Urban counties		
Original				
		1	1,527,211	3
		2	1,593,291	5
		3	1,507,456	2
		4	1,251,231	1
		5	1,552,508	4
	Range		342,060	
Average		1	3,165,367	4
		2	2,899,058	1
		3	3,221,790	5
		4	3,073,417	2
		5	3,087,596	3
	Range		322,732	

Table 16. (Continued)

Constraint		Plan number	Average deviation ($)	Rank
		Urban counties (continued)		
Maximum		1	3,974,896	4
		2	3,418,105	1
		3	4,066,091	5
		4	3,733,779	2
		5	3,770,898	3
	Range		647,986	
		Rural counties		
Original				
		1	134,571	3
		2	135,109	4
		3	99,731	1
		4	211,356	5
		5	123,599	2
	Range		111,625	
Average				
		1	185,168	3
		2	205,473	5
		3	186,860	4
		4	179,312	1
		5	184,850	2
	Range		26,161	
Maximum				
		1	310,539	3
		2	331,007	4
		3	333,338	5
		4	302,887	2
		5	279,184	1
	Range		54,154	

When only the urban counties are considered, the pattern established in the 21 county unit continues. Plans 2 and 3 remain the best and worst respectively in the solutions calculated under the average and maximum bounds. Under the original constraints, there is a change. The distribution of funds under plan 4 shows the least deviation from the optimum and that under Plan 2 shows the most deviation from the optimum.

For the rural counties, the pattern in the 21 county unit and the urban counties no longer holds. Plans 3, 4, and 5 show the least deviation from optimal under the original, average and maximum constraints while Plans 4, 2, and 3 are the furthest from optimal.

A comparison of the results of the quantitative analysis with the rank correlations provides evidence of the shortcomings of qualitative analysis. When the 21 county unit is considered, the results of the quantitative and qualitative analyses are the same for the original constraints and for the average constraints, but the parallelism ends with the third set of restrictions. The correlation between the optimal distribution and Plan 1 is the highest (0.73) whereas Plan 2 had the lowest dollar deviation. The correlation between the optimum and Plan 2 is 0.6685. This illustrates that fairly close rank correlations can conceal or distort absolute or dollar differences. This divergence between the rank correlations and the average dollar deviations is similar for the urban counties and even more substantial for the rural counties. For example, given the average bounds, the rank correlation between Plan 4 and Plan 0 is 0.56, the lowest of the correlations, yet Plan 4 showed the smallest dollar deviations ($179,312). Because the deviations are averages, it is possible that this measure caused distortions. One large deviation in a county would be enough to cause a large average deviation. This is a traditional problem in the employment of average measures.

In order to surmount the difficulties involved with the use of averages, it was decided to use a measure apropos of the programming approach. If a county were to receive an amount under a given plan less than the amount required under the optimal distribution the solution would be unfeasible. Hence the additional amount that must be added to a county's share in order to generate a feasible solution is called the programmed amount. This programmed amount was summed over all counties for each constraint. The results are summarized in Table 17.

Table 17. Additional amount necessary to meet varying constraints given the minimum optimal amount distributed according to alternative plans

Constraint	Optimal amount ($)	Plan numbers	Additional amount ($)	Rank
		21 counties		
Original				
	8,604,194	1	4,894,595	4
		2	5,069,093	5
		3	4,566,603	1
		4	4,784,555	2
		5	4,949,662	3
	Range		497,495	
Average				
	15,249,273	1	9,445,881	4
		2	8,756,326	1
		3	9,549,583	5
		4	9,118,647	2
		5	9,119,292	3
	Range		793,257	
Maximum				
	39,999,158	1	12,413,032	4
		2	11,208,318	1
		3	13,740,004	5
		4	11,759,391	3
		5	11,716,307	2
	Range		2,531,686	

Table 17. (Continued)

Constraint	Optimal amount ($)	Plan numbers	Additional amount ($)	Rank
		Urban counties		
Original				
	8,114,277	1	4,690,324	3
		2	4,902,847	5
		3	4,397,523	1
		4	4,566,603	2
		5	4,753,134	4
	Range		505,324	
Average				
	12,441,966	1	8,189,883	3
		2	7,772,879	1
		3	8,199,537	4
		4	7,991,791	2
		5	8,294,692	5
	Range		521,813	
Maximum				
	31,296,231	1	10,779,661	5
		2	9,607,685	2
		3	10,034,077	3
		4	9,263,694	1
		5	10,267,672	4
	Range		1,515,967	
		Rural counties		
Original				
	489,917	1	204,272	4
		2	161,251	1
		3	169,080	2
		4	217,952	5
		5	196,528	3
	Range		56,701	
Average				
	2,807,307	1	1,255,998	4
		2	983,447	2
		3	1,260,046	5
		4	1,206,856	3
		5	904,600	1
	Range		298,745	

Table 17. (Continued)

Constraint	Optimal amount ($)	Plan numbers	Additional amount ($)	Rank
		Rural counties (continued)		
Maximum				
	8,702,927	1	1,633,371	3
		2	1,600,632	2
		3	3,705,927	5
		4	2,495,697	4
		5	1,448,635	1
	Range		2,257,292	

The interpretation of these results is as follows. For the first constraint, the minimum amount necessary for a feasible solution is $8,694,194. (See Table 17, 21 counties, the second column). $4,566,603 must be added to the eight and one-half million already distributed according to this plan to make the program feasible. This is the smallest programmed amount.

Based on this measure Plan 3 is the most efficient (or at least inefficient relative to the optimum) of the five alternatives. For the second and third constraints, Plan 2 is the most efficient and Plan 3 is the least efficient. These results are consistent with the findings of the average deviation measure. For urban counties Plan 3 is the most efficient under the original constraints. Plans 4 and 5 are the most efficient distributions under the second and third constraints. This evaluation conflicts with the findings of the average deviation method and the rank correlations which indicate no differential output effects among any of the plans. In the rural counties, Plans 2 and 5 are the most efficient. These findings are also inconsistent with the deviation method and does not follow results closely with the correlation.

Output effects of migration

In order to simulate migration, the average daily membership in each of the rural districts was decreased ten percent and then increased the same proportion for urban districts. Since there were already more students in urban districts this caused a slight over all increase in student population from 13,407 to 13,655. To satisfy the original score constraints a state aid payment of $8,338,700.00 is necessary. This is a $265,494 decrease from the amount needed for the pre-migration distribution of students. The value of marginal product (0.07) remains the same. When the maximum score constraints were imposed, $19,019,200 was required to compute a feasible solution. This was less than one-half of the state aid ($49,999,158) needed under the original situation. The value of marginal product (0.05) is the same. These results imply that the migration will cause a decrease in the demand for state aid because the migration represented a movement of student inputs from less efficient educational processes to more efficient ones. There are two qualifications to these findings. First, this is a comparative static analysis subject to the limitations discussed on page 6. Second, there was a decrease in rural local spending and an increase in urban local spending resulting in a net increase of $3,367,650 in total educational expenditures within the state. It was assumed that no additional physical capacity was required in the five urban counties to absorb 658 new students (an average of 131 per county). This may not be a realistic assumption.

Summary and Conclusions of the Analysis

In conclusion, given a set of score constraints, the output effects of the alternative distribution plans are affected by the area in which they are applied. A plan may have superior output effects for a combined urban rural area, but may not be the best one when the rural or urban areas are considered separately. This means that if output maximization is the criteria by which plans are to be judged, the plan chosen will very likely be one that is not the best in every individual area. In the basic 21 county area designed to reflect the distribution and diversity student population of the State of Iowa, Plan 2, the sophisticated percentage equalizing plan was superior for two out of three sets of constraints as measured by the deviation and programming criteria. The rank correlations did not show substantial output effect heterogeneity. The output results of the plans were similar for the urban counties. For rural counties, there was no distinctly superior alternative. The output effects varied by constraint and measure (qualitative or quantitative) of association with the optimum. The effect of migration depends on the score constraint and the extent of student movement.

CHAPTER V. SPECIFICATION OF THE RELATION BETWEEN EDUCATIONAL OUTPUT AND INPUT

The analysis of the output effects of state aid as well as the systems analytic approach in general assumes that the relation between educational output and input can be specified. There have been problems encountered in this specification. These problems are discussed in this chapter.[1]

Introduction and Detailed Summary of Relevant Research

The Quality Measurement Project in New York State (29) represented the earliest systematic attempt to specify a relation between educational output and input. The average score on the Iowa Test of Educational Development (ITED) was used to measure output and was the dependent variable. The independent variables utilized to account for inputs and school system parameters were educational costs, percent of teachers having greater than five years of training, student IQ and socio-economic background. Educational cost was defined as expenditure per student in average daily attendance.

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[1] There have been numerous studies of the relation between educational output and expenditure related inputs, where a measure of the quality of the educational process was used as a proxy for a direct measure of educational output (i.e. instead of achievement tests). Many of these are summarized in (61). Other analysts have used income earned as a proxy (63). Welch (85) was the only one to relate the income measure to characteristics of the educational process inputs. He considered expenditure per student, teacher's salary, teacher-pupil ratio and size of school. Since the purpose of the research described in this study was to make decisions at the regional or local level, it was felt that a direct measure of educational output was required. For this reason achievement test scores were used instead of income. It was thought that this would eliminate the effect of the vagaries of the labor market.

The importance of socio-economic background in predicting achievement was the major finding. When the effect of socio-economic status was considered, the correlation between expenditure and achievement was found to be 0.31 (significant at the five percent level). The correlation between expenditures and achievement increased with the level of socio-economic status as did the correlation between achievement and teacher background. The authors of the study concluded that there is a determinate relation between expenditure and achievement, but it is blurred because schools tend to use additional financial resources to benefit pupils of higher socio-economic status at the expense of others (29, p. 46). Thus, the specific distribution of the additional funds within the systems is important.

Following the lead of this New York State Quality Measurement Study, Kershaw and McKean (42, p. 17) advocated the use of covariance analysis to specify the relations between educational output and educational inputs so that costs and benefits of alternative educational policies could be evaluated. They were among the first to advocate application of the systems analytic approach to elementary and secondary education. Kershaw and McKean suggested using covariance because they thought it would be the best way to eliminate the effect of interdependencies among the parameters (i.e. such as between pupil I.Q. and home environment). They acknowledged the possibility that the parameters or control variables might be of such importance that they would swamp the effects of the variable school inputs, but they did not believe it to be serious.

Thomas' study (77) focused on the factors emphasized by Kershaw and McKean (42). Using 1962-63 Iowa data, this study employed multiple regression techniques to estimate effects of six categories of expenditure (general control, instruction, auxiliary, operation of plant, maintenance and fixed charges) on average school ITED scores while accounting for the attitude of the school district toward education, the economic class of the county, the size of the school, the breath of the curriculum and the quality of the teachers. Thomas found a small but statistically significant relationship between level of achievement and per pupil expenditure. Both the partial regression coefficient of achievement and expenditures and the R^2 increased for small (average daily attendance 0-245 and 246-422) Iowa School districts when the districts were classified by size of student body.[1] The R^2 was 0.14 and 0.40 for these groups. This study also investigated the relationship between output (ITED) and teacher characteristics (college training) and work load (assignments per teacher). There was a small, but significant negative relation between output and average number of assignments per high school teacher. Unfortunately, this equation explained only 0.07 of the variance.

In view of the rather limited explanatory power of expenditures, Cohn (14) drawing upon the Thomas sample, did not consider expenditures per student in his models designed to analyze educational quality where quality was again based on district average test scores. He considered the determinants of expenditure per student separately. Cohn included in his quality

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[1]Of the 366 school districts out of the possible 510 in Iowa, the districts analyzed ranged from less than 116 students in average daily attendance to 12,323. The distribution had modes at 423-493 and 765-933.

models such cost related items as semester hours of college per high school teaching assignment, assignments per teacher, median teacher salaries, building value per pupil, plus geographic and population variables included as dummy variables. Cohn found small but significant positive relationships between the per pupil achievement index and semester hours of college education per teaching assignment, and between the index and teacher salaries. He found a small negative relation between the achievement index and the value of building per student.

Cohn's linear models like Thomas' and those of the New York State Quality Measurement Study, indicated that a relatively small amount of variance in output was explained by cost factors. The use of multiplicative (logarithmic) models did little to increase the R^2. When the achievement score of the previous year was included, the R^2 increased substantially above fifty percent. The simple correlation between present and previous test score is 0.79 (14, p. 95). This made the propriety of its inclusion questionable. There is less objection to this procedure where Cohn is considering changes in test score as his dependent variable. Another finding of the Cohn study (illustrative of problems of coefficient specification in education systems) was the nonsignificant relation between district average cost per pupil and instructor's salaries. This nonsignificance occurred both in the simple correlation (14, p. 95) and in a multiple regression equation (14, p. 103).

Drawing upon New York State Quality Measurement Project sample, Kiesling (43; 44), in measuring government service, did not find a significant relation between average high school district achievement test scores and expenditure per pupil by testing relationships first within a linear context, then within a nonlinear context. The inclusion of IQ scores and a dummy variable for effect of grade did enable his equations to explain a significant amount (0.81) of the variance in test scores (44, p. 361). When stratifying the sample according to occupation of father, Kiesling found expenditure per pupil to be significant for some of the socio-economic categories in the primary school level (43, pp. 34-38). In urban areas with more than 2,000 pupils in average daily attendance (ADA), expenditure per pupil was significant in all but the unskilled workers category (43, p. 53). Kiesling summarized by saying:

> After reasonable allowance is made for sociological and intelligential differences on the part of pupil populations, the relationship of performance to pupil expenditure has been found to be . . . disappointedly weak. This would imply, among other things, that the utilization of per capita cost figures for an index of public performance is a highly dangerous practice (44, p. 366).

The importance of looking at both educational outputs and inputs for assessing state aid expenditures has been stressed in this study. However, to bring analysis from regional levels to a school district level it is very desirable to specify a reliable functional relationship between educational output and educational costs. In the previous chapter using systems analytic study of state aid plans for a region, one point on each district's production is sufficient. However, for one district a different analysis is needed. Some writers see hazards in using educational expenditures as an index to evaluate an individual school system's output, while specification of a relationship between educational outputs and inputs so necessary for analysis remains to be accomplished. Attempts to fill this void are surveyed in this chapter.

A survey of educational opportunity,(hereafter to be referred to as "Survey") directed by James Coleman (17), was the most comprehensive study of the nation's educational system to that time. The purpose of this Survey was to determine the effect of racial and ethnic discrimination in the schools on educational opportunity. When the study was completed in the summer of 1966, data on superintendents, principals, teachers and 645,000 pupils collected from a nationwide sample of 3100 schools were analyzed to determine:

> (1) the extent of racial and ethnic segregation; (2) the degree of inequality in the provision of school resources among racial and ethnic groups; (3) the performance levels of students of different backgrounds on achievement tests; and (4) the relationships between school and student characteristics on the one hand and students' achievement on the other (11, p. 4).

Findings on relationships between school and student characteristics of the report were similar to those of other studies (17, pp. 325-333). That is:

> (1) Per pupil expenditures, books in the library and a number of other facilities and curricular measures show very little relation to achievement if social background and attitudes of individual students and their schoolmates are held constant.
>
> (2) The effect of a student's peers on his own achievement level is more important than any other school influence (17, p. 325).

The multiple regression method of relating average school district achievement to expenditures and to the other factors was employed in the Survey. Coleman, the principal author of the Survey, emphasized the importance of testing the significance of the additional variance of achievement scores that is explained by an independent variable such as expenditures, whereas Thomas, Cohn and Kiesling concentrated on the estimation of partial regression coefficients. Coleman thought that exploratory research required the determination of which were the relevant factors rather than attempts at coefficient specification. His procedure with respect to investigating the achievement impact of expenditures (and of facilities and services procured by these expenditures) was to determine how much additional variance in achievement was explained after accounting for non-school characteristics (16, p. 328).

In the Survey, correlation between achievement and expenditures was calculated for each grade. If expenditures had a cumulative effect, it was hypothesized that the correlation coefficient would increase by grade. It did not (16, p. 311). It was reported that the lowest achieving minority groups showed the highest sensitivity to differences in public school characteristics most closely related to expenditures. Setting the stage for the Survey's major findings on aspects related to the effects of school expenditures on achievement, it was stated that because between school achievement variance was less than within school variance at all grade levels and for all racial and ethnic groups, "most of the variation could not possibly be accounted for by school differences" (17, p. 297).

Because the systems analysis approach to the micro-economic problem of allocation of educational resources requires definition of input-output relations, economists and educational administration specialists have criticized the findings of the Survey. Gavin and Spitzer (28) attempted to amend the Survey's findings with respect to the expenditure related instruction variables by using partial correlations. Holding school mean verbal score constant, they estimated the association between student verbal achievement score and per pupil instruction expenditure. They were not successful in modifying the Survey's conclusions; 0.08, for Spanish speaking Americans was the highest coefficient calculated (28, p. 15).

Bowles and Levin were the severest critics of the methodology used in the Survey. They argued that the achievement test score distribution of nonrespondent schools affected the Survey's expenditure findings (11, p. 6). Further, the measures of school resources used (school district instructional expenditures per student, facilities per student and pupil-teacher ratios) did not reflect the actual differences among students within a school in resources allocated per student. Bowles and Levin further indicated that since the Survey used district expenditures per student, systematic bias in resource distribution between schools within a system could distort the relationship between achievement and expenditures (11, p. 8).

Using district average scores reduces the variance in test scores that could potentially be accounted for by school expenditure variables argued Bowles and Levin. They hypothesized that the positive relationship between teacher salaries and achievement and the negative relationship between expenditures per pupil and achievement was caused by the averaging of per pupil expenditures over the school district while the teacher's salaries averaged over the school (11, p. 9). With respect to differences in expenditures per pupil within schools, a positive correlation between teacher salaries and the pupil teacher ratio would rationalize the negligible effect of expenditures on achievement. In most cases, the correlation reported in the Survey was negative which, Bowles and Levin concluded, tended to support the hypothesis of a positive relation between achievement and expenditures (11, p. 10).

Bowles and Levin's most serious methodological criticism was on the manner in which the effect of expenditures and expenditure related variables on achievement was isolated. They argued that unless the explanatory variables were perfectly independent, the order in which the explanatory variables were introduced into the equation was important to the analysis of the variable's contribution to explained variance. This was because when explanatory variables are related, the explanatory power they share will be attributed to the variable first introduced. Since background of students and school expenditure characteristics were related, and the Survey introduced student background first, the explanatory power of student background was over-stated and the explanatory power of expenditure was understated (11, p. 14). Because the multiple regression technique does not account for time sequence, there was no justification for putting in student background variables first, according to Bowles and Levin.

Replying to Bowles and Levin, James Coleman explained that in the Survey, they did attempt to account for nonresponses, especially by schools at the low end of the achievement distribution by using multiple indexes of socio-economic class (16, p. 238). Coleman admitted that the Survey did not always distinguish between the school and the school system, but he argued that it was possible to determine achievement effects of expenditure by analyzing variance in school system achievement scores that could be explained by system differences in school system input expenditure (16, p. 239).[1] Coleman reported that the relation between the student background variables and expenditures was not close enough to violate the orthogonality (independence) assumptions of multiple regression. Thus, the procedure of analyzing the additional variance explained by the expenditure variable after accounting for that explained by sociological variables was valid. This author would have preferred to see the correlations and judge for himself. However, Coleman did indicate agreement with Bowles and Levin that additive linear multiple regression is only one of many possible methods and there is room for the use of other models and measures in exploring the relation between achievement, expenditures and student background factors (16, p. 240).

Diagnostic Hypotheses Concerning the Specification of Output-Input Relationships

The basic criticisms of the Survey also applicable to the other studies cited, are in effect explanations or hypotheses as to the causes of the problems of specifying relationships between educational output and expenditures. Besides the sampling issue other criticisms focused on the measurement of the variables, appropriateness of the statistical estimation model and procedures followed when using the model. Because of the significance and of the criticisms discussed efforts to test three diagnostic hypotheses are now reported. These hypotheses are:

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[1]The Survey often used the term "school" when it meant "district." See 17, p. 330 for an example.

(1) Does achievement-cost data conform to the assumptions of the regression model? (2) What are the effects of alternate measures of the variables (i.e. average versus individual observations) within the context of the general linear model? (3) What is there to be gained by using factor analysis as an alternative statistical instrument that focuses upon hypothesis searching or on the existence of relationships rather than on the magnitude and direction of relationships?

The heteroscedasticity hypothesis

One of the assumptions of the least squares techniques used in regression analysis is that of homogeneous variance. For the relation between achievement and expenditure, this implies that for each level of expenditure, there is a distribution of achievement test scores such that the variance of each distribution is the same (37, p. 355). If the achievement variance is positively correlated with expenditure level, the estimate of the derivative of achievement with respect to expenditure (the regression coefficient) would be understated. If the achievement variance is negatively correlated with expenditure level, the reverse is true (37, p. 375). Heteroscedasticity, the absence of homogeneous variance, perturbs the estimation of the standard error of the regression coefficient and hence distorts the test of significance (37, p. 363).

Because of the importance of the homoscedasticity assumption in the specification of least square relationships, it was the first item investigated. The individual student 12th grade ITED (Iowa Test of Educational Development) Scores for 1962-63 were obtained for each of the schools in the twenty-one counties of the basic sampling unit (60). The expenditures per student (1962-63) in each school were obtained (77). A frequency distribution was constructed. There were 104 different cost figures. They were arbitrarily divided into five groups. In other words, the expenditures were grouped from highest to lowest. The first twenty highest expenditure levels per student were put in group one and the lowest level of expenditures were put in group five. There were 7720 students in all of the schools. It was decided to take a five percent (386) sample. The number of students taken from each school was determined by the school's percent of the 7720 students. The random number table was used to decide which individual student score was picked from each school. The next step was to determine the variance within each of the five expenditure groups and to test for homogenity of variance using Bartlett's test (66, pp. 285-289).

Score means along with the score standard deviation and the cost means and cost standard deviations for each expenditure group are presented in Table 18. Note that the students are also analyzed into rural and urban groups.[1] For the entire sample of students, the score variances between the expenditure groups are similar. This holds for the rural students, but not for the urban students. Note that the positive correlation between expenditure variance in the urban group would understate a regression coefficient. For the total group and the rural group a one way analysis of variance was performed to ascertain significant score differences among the expenditure categories. See Table 18. For the total group, expenditure category makes a difference in average score. In the rural group, there is no significant difference. A special test of significance (66, p. 208) applied to the urban data indicates a measurable expenditure effect.

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[1]The urban place students separated in the analysis reported in Chap. IV are included with the urban group.

Table 18. Sample 1962-63 statistics on achievement test scores and expenditures, and results of significance test of expenditure variation on score (computed from 60 and 77)

Sample	Sample size	Group	Expenditure per student: Mean ($)	Expenditure per student: Standard deviation ($)	Achievement test score: Mean	Achievement test score: Standard deviation	Test statistic
Total	98	1	334.388	18.456	19.786	5.962	F = 4.199*
	67	2	362.000	7.687	19.626	5.507	
	67	3	391.716	8.913	22.493	6.292	
	118	4	437.144	15.482	21.322	6.568	
	36	5	505.083	69.560	18.083	7.133	
Urban	66	1	341.379	10.961	20.000	5.860	X^2 = 89.623*
	25	2	363.120	8.634	17.800	5.196	
	41	3	389.561	7.846	24.268	6.090	
	90	4	437.589	15.617	21.767	6.564	
	8	5	545.250	136.670	19.625	8.088	
Rural	32	1	319.969	22.254	19.343	6.240	F = 1.064
	42	2	361.333	7.087	20.714	7.087	
	26	3	395.115	9.572	19.692	5.712	
	28	4	435.714	15.226	19.893	6.494	
	28	5	493.608	28.558	17.642	6.935	

*Significant at five percent level.

The measurement hypothesis: individual versus averages

The use of school district averages in measuring achievement and the use of school district average measures of resources per student in analyzing the relation between educational output and input has been criticized. Abel and Waugh (1) discussed the relationships between group averages and individual observations in linear regression. They demonstrated that the simple regression coefficient for individual observations may be analyzed into between group regression (the regression obtained from using averages) and within group regression. That is:

$$b_{yx}(\text{ind}) = \frac{\Sigma \bar{x}\,\bar{y} + \Sigma\,(x-\bar{x})(y-\bar{y})}{\Sigma \bar{x}^2 + \Sigma\,(x-\bar{x})^2}$$

$$\text{Between group regression} = \frac{\Sigma\,\bar{x}\,\bar{y}}{\Sigma\,\bar{x}^2}$$

$$\text{Within group regression} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

$$b_{yx}(\text{gr}) = \frac{\Sigma\,\bar{x}\,\bar{y}}{\Sigma \bar{x}^2}$$

Since b_{yx}(ind) represents an average of within and between group regressions, it "will be greater than, equal to, or less than the group regressions depending upon whether the regressions within groups are greater than, equal to, or less than the values of the regressions <u>between</u> groups" (1, p. 112). In the examples presented by Abel and Waugh, grouping by the x variable and regressing the averages yielded (1) a slightly lower regression coefficient then when unweighted averages were used and (2) the same regression coefficient as individual, ungrouped data when weighted averages, were used. Thus, with the unweighted averages, the within group coefficient was lower than the between group coefficient and the within group coefficient was the same as the between group coefficient for the weighted averages.

The results of regressing score on expenditure using the sample described above are presented in Table 19. Although none of the regression coefficients are significant at the five percent level, it is of interest to observe the behavior of the individual versus average measures. The regressions were computed for the total and rural data group which had homogeneous variances between expenditure groups. For the total sample, the regression coefficient for the individual observations is a very small negative (-0.0001). When the observations are grouped by expenditure and average achievement is regressed on average expenditure the regression coefficient remains negative, but increases (-0.0002). It can be seen that the lower individual regression coefficient can be explained by the low and sometimes negative within group regression coefficients. This pattern is repeated in the rural section of the sample.

Due to the formulation of the correlation coefficient $r_{yx}=(S_x/S_y)b_{yx}$ where S_x and S_y equals the standard deviation of X and Y, respectively, and b_{yx} is the regression coefficient, the transformation of data from individual observations to group averages affects the value of r_{yx} and its close relative r^2, the percent of variation explained by regression. "r^2" is usually expressed as "R^2". The process of averaging by groups does not

Table 19. Regressions of achievement on expenditures per student - 1962-63 (computed from 60 and 77)

Sample	Level of aggregation (test score)	Expenditure group (if applicable)	Sample size (n)	Regression coefficient (B_{yx})	Coefficient of determination (R^2)
Total	Individual		386	-0.0001	0.0001
	Average (Grouped by expenditure)		5	-0.0002	0.0003
	Within group	1	98	-0.0059	0.0003
		2	67	0.0179	0.0006
		3	67	-0.0286	0.1049
		4	118	0.0014	0.0001
		5	36	0.0033	0.0001
Rural	Individual		156	-0.0091	0.0084
	Average (Grouped by expenditure)		5	-0.0697	0.1124
	Within group	1	32	0.0397	0.0193
		2	42	0.1293	0.0328
		3	26	0.0148	0.0006
		4	28	-0.0533	0.0156
		5	28	0.0018	0.0000

always result in equiproportional reductions in the standard deviations of y and x (S_y and S_x), (1, p. 111). The averaging process when grouping by expenditures reduced the S_x relative to S_y such that the R^2 increased from 0.0001 to 0.0003. The change in the R^2 for the rural group is more dramatic (0.0084 to 0.1124). The same effect was noted in Abel and Waugh's examples (1, p. 111).

What are the implications of these findings for interpreting the measurement criticisms directed at the studies cited above? On an a priori, theoretical basis, the use of averages can lead to stronger or weaker regression relationships than the use of individual data. If the education data used in the studies referenced in this report had characteristics similar to that used in this study, it can be inferred that by employing averages, they achieved the highest possible regression and correlation coefficients. Perhaps, even the use of school data as distinguished from school district expenditure data is too high a level of aggregation to obtain significant input-output relationships.[1] This is a logical extension of Bowles and Levin's (11) measurement criticism. The divergent within group relations are testimony to the highly variable results from the same amount of dollar expenditure. The simple regression findings presented here should not be taken as support for the hypothesis that there is no relation between expenditures and achievement because third factors that are excluded might be causing the relationship in this set of data to be distorted. Perhaps the groupings were not satisfactory, because the regression model assumes there is no distribution on the independent variable. As can be seen

[1] Other researchers using measures of process quality as proxies for output achieved significant relations between those measures and expenditures for schools (61, pp. 609, 612, 614, 618, 620).

from Table 18, there is some variation of expenditure within each group. The next step of the analysis was to explore factor analysis, an alternative statistical technique directed at determining which factors were most important in determining educational output on the proposition that at some future time it would be feasible to trace out the expenditure implications of these factors.

The factor hypothesis

Factor analysis is "a method of analysing a set of observations from its intercorrelations to determine whether the variations represented can be accounted for adequately by a number of basic categories smaller than that with which the investigation was started. Thus, data obtained with a large number of *a priori* measures may be explained in terms of a smaller number of reference variables" (58, p. 1). The factor analytic approach is, "predicated on the assumption that the original variables introduced in a multivariate analysis exhibit largely superficial manifestations of true relationships, and that beneath these superficial relationships are certain factors [reference variables] common to various [linear] combinations of these variables. . ."(25, p. 101). These factors are hypothetical constructs. Factor analysis has been used extensively in psychology and education in order to determine underlying determinants of test performance (41, p. 650). It has not been used extensively in economic analysis. Adelman's macro-economic oriented factor analytic study of the relationship between social and political variables and gross national product per capita (4) and Oehrtman's micro-economic oriented analysis of the problems facing milk bottling firms (56) are recent examples of the use of factor analysis in general economic studies. Mayeske (50) and Ayer in Ross (61) employed factor analysis in studies related to the input-output specification problem which is of principal concern in this chapter. The outline for the remainder of the chapter is as follows. First, the factor model will be briefly discussed. Second, the Mayeske and Ayer studies will be summarized. Third, the results of a factor analytic study employing the 1962-63 Iowa Data described earlier in this chapter will be examined. Finally the implications of the factor analyses to the specification problem will be summarized.

Drawing upon several sources, Oehrtman rigorously outlined the factor model. Much of what follows leans heavily on his work (57). The classical factor analysis model is of the form:

$$Z_j = a_{j1}F_1 + a_{j2}F_2 + \ldots + a_{jm}F_m + d_jU_j \quad (j=1,2,\ldots,n)$$

where each of the n observed variables is described in terms of m, $(m < n)$ common factors and one unique factor. The common factors account for the correlations among the variables, while each unique factor accounts for the remaining variance (including error) of that variable. A common factor is involved in more than one variable of a set and a unique factor is involved in a single variable of a set (58, p. 12). The a_{ji}'s, or "factor loadings" may be interpreted in three ways. First, they can indicate the degree to which variables are related to each factor, in a manner similar to a net correlation coefficient (the values range from -1 to +1); the square of the factor loading is the percent of variance of the variable that is explained by the factor. Summing this quantity across all factors results in the percent of variance of the variable explained by all the factors. Second, the loadings indicate the relative importance of each factor in acting upon the variable. Third, the loadings indicate a basis for combining variables into common groups. Variables are grouped according to their factor loadings. The set of variables having high loadings in a particular factor are said to be members of the same group. Therefore, a group is defined as a set of variables having high loadings in a given factor. Presumably, there is some common bond among all of the members of a particular group.

Interpretation two is possible because the variance of an individual variable may be expressed in terms of the sum of the squares of the common factor coefficients. Hence, the communality of a variable Z_j is expressed:

$$h^2_j = a^2_{j1} + a^2_{j2} + \dots a^2_{jm} \quad (j=1,2,\dots,n)$$

and the uniqueness, the extent to which the common factors fail to account for the total unit variance of the variable is expressed:

$$d^2_j = 1 - h^2.$$

The assumptions of the factor model should be noted:

1. the model is linear
2. $E(F) = E(U) = 0$
3. $E\ (FF') = I$, the common factors are uncorrelated and have a variance equal to 1
4. $E(UU') = V$, a diagonal matrix
5. U is distributed independently of F, and both F and U have multivariate distributions
6. The Z's have a multivariate normal distribution.

It should be recognized that there is no unique set of factors. Given a correlation matrix it can be described by an infinite set of factors or reference axes. This is demonstrated by noting that given points or correlation coefficients mapped on Cartesian space, the y and x axes can be rotated to intersect or come close to the given points. There are any number of possible rotations that fit the points to the same degree (12, p. 668). Any correlation may be referenced by an infinite number of orthogonal vector systems. Consequently, an infinite number of factor loadings is possible. The use of factor analysis has been criticized based upon this indeterminancy of the factor loadings (25, p. 107).

However, the indeterminancy can be used for the benefit of the researcher because this characteristic aids in reducing the complexity of a factor solution in that it increases the simplicity of interpreting the factor solution. This simplicity of interpretation is achieved without altering the constellation of factor correlations. Hence, this criticism is unfounded.

To aid the factor analyst in judging his rotation configuration Thurstone in Kerlinger (41) has established the following principles:

1. Each row of the factor matrix should have at least one loading close to zero.
2. For each column of the factor matrix, there should be at least as many variables with zero or near-zero loadings as there are factors.
3. For every pair of factors (columns) there should be several variables with loadings in one factor (column), but not in the other.
4. When there are four or more factors, a large proportion of the variables should have negligible (close to zero) loadings on any pair of factors.
5. For every pair of factors (columns) of the factor matrix, there should be only a small number of variables with appreciable (nonzero) loadings in both columns (41, p. 669).

Kerlinger summarized the meaning of the criteria as calling for as "pure" a factor as possible such that each variable is loaded on as few factors as possible, and as many zeros as possible appear in the rotated factor matrix. These criteria specify a way to achieve the simplest possible interpretation of the factors. "In other words," states Kerlinger, "rotation to achieve simple structure is a fairly objective way to achieve variable simplicity or to reduce variable complexity" (41, p. 669).

The methods used for determining the number of factors has also been criticized (25, p. 107). However, the determination of the number of factors rests on a sound mathematical base. One method of determining the number of factors is related to the internal process of computing factors. The process of computing factors involves maximizing the variance of the variables to be explained by the first factor (communality). This variance is maximized subject to the constraint that the original correlation matrix can be reproduced from the factor loadings (i.e. the residual between the original correlation matrix and that correlation matrix generated by the factors approaches zero). By using Lagrange multipliers a set of equations is determined and equated to zero (31, p. 155). These equations are analagous to the "normal equations" of multiple regression. The coefficients of the equations are written in the determinant form. This is called a "characteristic equation" (31, p. 157). The largest root of this equation is substituted back into the "normal equations" from which the factor loadings of the first factor are obtained. The roots of a "characteristic equation" are called the "eigenvalues". Each "eigenvalue" will generate a new set of factor loadings. Since multiplication of the square root of the "eigenvalue" is involved in obtaining the factor weights, factors can be generated until the "eigenvalues" approach zero. Hence, it is possible to judge from the "eigenvalues" or their first differences whether additional factors will explain additional variance. Initially, it is decided to abstract an arbitrary amount of factors, but only the "significant" number of factors are kept. When the "eigenvalues" begin to approach zero asymptotically the process of generating factors ceases (31, p. 187). Recently, a chi square test for factor significance has been developed (31, p. 372). However, the analysis of the "eigenvalues" continues to be the best initial estimate for the number of factors to be extracted from the original correlation matrix (59).

Ayer in Ross (61) was interested in the relation between measures of the quality of the educational process (the Growing Edge which is based on the presence or absence of certain educational practices and the Time Scale which is related to when the school adopts educational innovation) (61, pp. 27-28), and community population characteristics, size, tax practices and expenditure per pupil. Ayer's factor analysis is presented in Table 21. F_i, i = 1,5 represents the ith factor.

Expenditures per pupil (variable 21) has a high loading in Factor one; h^2 is equal to .80 which indicates that 80 percent of the variance in expenditures per pupil is accounted for by the first factor. Other variables in this factor are wealth variables (13 and 14), tax variables (16-19) and the process quality measures (variables 21 and 22). Of the 50 percent and 95 percent of the variance of variables 21 and 22 explained by the resolution

Table 20. Community analysis factor loadings from metropolitan school study council data, 1940-46, computed by Frederick L. Ayer in Ross (61, p. 624)

		F_1	F_2	F_3	F_4	F_5	H^2 (highest)
1.	Percent of population eighth grade graduates		-0.95	-0.29			0.90
2.	Percent of population college graduates		-0.49		0.26		
3.	Percent of population unskilled		0.79				
4.	Percent of population business and professional		-0.87				
5.	Percent of population foreign born		0.44		0.34	-0.26	
6.	Percent of population 50 years				-0.38		
7.	Percent of population own home		-0.46				
8.	Opinion poll		-0.47				
9.	Area of school district		0.48			0.45	
10.	Total enrollment			0.68		-0.54	
11.	Total population of district					-0.63	
12.	Density of population		0.31	0.80		-0.53	
13.	Wealth of school district	0.99				-0.35	
14.	Wealth trend	-0.47		-0.39	-0.32		0.99
15.	Assessed true value	0.37			0.72		
16.	Total tax rate	-0.60			-0.54		0.84
17.	School tax rate	-0.67			-0.67		
18.	Percent of school tax debt serviced				0.27		
19.	Tax leeway	0.76					
20.	Expense per pupil	0.88					0.80
21.	Time scale	0.31		0.60			0.50
22.	Growing edge	0.58		0.50	0.31	0.31	0.95

into factors, F_1 explains 10 percent and 34 percent respectively. For the Growing Edge (variable 22), F_1 explains the most variance, while F_3 explains most of the variance of the Time Scale (variable 21). If the Growing Edge Process Measure were considered to be a proxy for an educational product measure, it can be concluded that there is some linear relation between expenditures and quality of educational output (as well as wealth). In attempting to define the underlying elements in his analysis of education and the community, Ayer labeled his factors as follows: F_1, total community setting; F_2, population characteristics; F_3, urbanness; F_4, tax practices; and F_5, population scantiness. Expenditures per pupil shows up only in F_1. In evaluating Ayer's results, it can be noted that all of Thurstone's principles of factor rotation are fulfilled with the possible exception of F_3. There is some overlap of four of the variables comprising F_1 and F_4. However, on _a priori_ basis, it would be expected that tax practices would be related to wealth and since there is no loading of the educational process variables in F_4, this nonfulfillment of Thurstone's third principle should not be interpreted as serious detraction from Ayer's findings of a relation between expenditure per student and an indirect measure of educational output. Also, Ayer's results are consistent with regression and correlation studies reported in Administration for Adaptability (61).

The relation between process variables (as a measure of product quality) and expenditure has been reported to be significant and positive (61, pp. 612, 614, 618, 620).

Mayeske (50) attempted to delve into the relation between expenditure related process variables and achievement. Instead of an overall measure of the process such as the Growing Edge, Mayeske considered characteristics of teachers and administrators. He used data collected in the Survey (50, p. 1). Criterion scaling was used to estimate missing data.[1] Mayeske conducted a factor analysis on the intercorrelations of five ninth grade achievement measures.[2] He concluded that a single factor describes the intercorrelations of these achievement measures. The students' scores were then weighted by the factor loadings and the average was used for each student's composite achievement test score. A set of student teacher and principal variables were also factor analyzed into composite indices. The indices and their components are listed in Table 21. Using the school as his basic unit (51, p. 6) Mayeske regressed achievement against the student body and school variables (broadened to include the teacher and principal variables listed in Table 21). An R^2 of 0.76 between the school variables and achievement scores was determined (50, p. 19). However, Mayeske also noted that the school variables were correlated with student variables (50, p. 18). Mayeske indicated that when the variance in achievement is first corrected for student variables, the contribution of school variables in explaining achievement decreased to five percent (50, p. 19). When the characteristics of the student body were held constant, the school's greatest measurable effect was on student expectations (ten percent of variance ex-

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[1]In criterion scaling, a missing item is filled in by assigning the mean value of the dependent variable for each of the different response alternatives for an item (5, p. 4). In other words, if an individual has an achievement score missing he was given the score necessary to make the average of all the various types of students equal.

[2]This portion of the analysis was based on work Mayeske had done with Weinfeld (51).

Table 21. Student, teacher, principal and school indices constructed by Mayeske (50)

Variable number	Students	Teachers	Principals and schools
1.	Expectations for excellence	Experience	Experience
2.	Socio-economic status	Teaching conditions	Training
3.	Attitude toward life	Localism of background	College attended
4.	Family structure and stability	Socio-economic background	Sex
5.	Educational desires and plans	Training	Plant and physical facilities
6.	Study habits	College attended	Instructional facilities
7.	Racial ethnic differences in achievement	Teaching related activities	Specialized staff and services
8.		Preference for higher ability students	Tracking
9.		Sex	Testing
10.		Racial ethnic differences in contextual vocabulary	Transfers
11.		Vocabulary score	Remedial programs
12.			Free milk and lunch programs
13.			Accreditation
14.			Age of texts
15.			Availability of texts
16.			Age of buildings
17.			Pupils per room
18.			Pupils per teacher
19.			Number of students enrolled in school
20.			School's reputation

plained) rather than on achievement.

Mayeske (50) felt that the small contribution of the school variables to explaining achievement indicates the difficulty of specification of the relationship between input and output rather than absence of relationship (50, p. 11). When he investigated the correlation between individual school variables (those comprising the index), the relation between these individual variables and achievement variables was found to be even smaller. Thus, he concluded, that his analysis indicated that small changes in just a few variables would not cause substantial gains in achievement and that perhaps only radical departures from current procedures would show measurable results[1] (50, p. 12).

Ayer used factor analysis to relate cost and qualitative measures of the educational process (indirect measures of educational product) within the community setting. Mayeske used factor analysis to relate quantitative elements of the education process with achievement (a direct measure of educational product) while adjusting for student differences. The factor analysis reported below was performed on achievement test scores, (educational output measures) one of which was adjusted for student differences, some measures of community attitudes and ability to pay for education, some indirect measures of the quality of the educational process itself (teacher salary) and seven categories of educational expenditure per student. All of the 510 school districts making up the 1962-63 Iowa data base described previously were used with the addition of 1960-61 scores to compute the change in score. The majority of the districts have one high school per

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[1]To test this the relation between the Growing Edge and Time Scale and Achievement should be tested.

district. The seventeen variables are listed in Table 22.[1] The simple correlation matrix is presented in Table 23.

In order to estimate the number of factors to extract the "eigenvalues" were computed. All "eigenvalues" were non-negative. The "eigenvalues" and their first differences appear in Table 24. Note that after the fourth "eigenvalue" the quantities tend to level off and asymptotically approach zero. This can be noted from the first differences which are approaching zero faster than the "eigenvalues" themselves. Four factors were initially extracted. Then, the general factor (F_1) was extracted from the four by four matrix of factor loadings formed by the first four factors (F_i, $i = 1,4$). The rotated solution of the factor loadings is shown along with the communalities (h^2_j) and unique variances (d^2_j) are shown in Table 25).

The rotation obtained conforms fairly well to the principles set forth by Thurstone. Factors two and three are not as "pure" as might be desired. Using 0.2 as an arbitrary cutoff point, there are eight instances of a variable appearing in both F_2 and F_3. However, in most of the instances the loadings of the pairs of variables are either opposite in sign or of sufficient difference in magnitude to determine in which factor a variable belongs.

It should be recalled that the primary emphasis is on the exploration of the relation between output of the educational process and expenditure related inputs. The output measures are variables X_{15}, X_{16}, and X_{17}.

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[1]Information in Tables 22 through 25 also appears in (57).

Table 22. List of variables to be factor analyzed

Variable	Description
X_1	Percent of county general fund devoted to public education
X_2	Number of high school units short
X_3	Number of high school units offered
X_4	Median family income in the county in dollars
X_5	Code size of average daily attendance
X_6	Average salary of elementary teacher in the school district in dollars
X_7	Average salary of high school teacher in the school district in dollars
X_8	Per pupil building value of the school district in dollars
X_9	Per pupil spending on general control in dollars
X_{10}	Per pupil spending on instruction in dollars
X_{11}	Per pupil spending on auxiliary services in dollars
X_{12}	Per pupil spending on the operation of plant in dollars
X_{13}	Per pupil spending on maintenance in dollars
X_{14}	Per pupil spending on fixed charges in dollars
X_{15}	The Iowa Test of educational development composite score for seniors for 1962-1963
X_{16}	The change in the Iowa Test of educational development composite score from 10th to 12th grades for 1960-1961 to 1962-1963 school years
X_{17}	Ratio of change in ITED composite score to change in ITED vocabulary score from the 10th to 12th grades for 1960-1961 to 1962-1963 school years

Table 23. Correlation matrix of variables to be factor analyzed (N=510)

Variable	1	2	3	4	5	6	7	8	9
1	1.000								
2	0.121	1.000							
3	-0.188	-0.676	1.000						
4	0.149	-0.063	0.230	1.000					
5	-0.176	-0.709	0.809	0.162	1.000				
6	-0.065	-0.497	0.655	0.365	0.638	1.000			
7	-0.088	-0.445	0.602	0.254	0.593	0.712	1.000		
8	0.031	0.054	-0.035	0.004	-0.052	0.078	0.146	1.000	
9	0.146	0.369	-0.494	-0.096	-0.563	-0.407	-0.364	-0.010	1.000
10	0.081	0.024	0.004	-0.026	-0.229	0.121	0.200	0.318	0.253
11	-0.055	0.280	-0.471	-0.307	-0.462	-0.474	-0.363	-0.006	0.424
12	0.067	0.295	-0.262	-0.045	-0.423	-0.192	-0.107	0.408	0.343
13	-0.137	0.057	0.000	-0.056	-0.121	-0.018	-0.020	0.081	0.094
14	-0.088	-0.188	-0.157	-0.139	-0.273	-0.124	-0.127	0.114	0.194
15	0.125	-0.129	0.131	0.094	0.200	0.212	0.282	0.061	-0.131
16	0.115	-0.159	0.136	0.091	0.187	0.125	0.135	-0.010	-0.103
17	0.078	0.006	-0.043	-0.003	0.003	-0.000	-0.034	-0.057	0.113

10	11	12	13	14	15	16	17
1.000							
0.199	1.000						
0.545	0.356	1.000					
0.247	0.133	0.218	1.000				
0.255	0.248	0.391	0.179	1.000			
-0.082	-0.215	-0.084	-0.030	-0.052	1.000		
-0.058	-0.143	-0.112	-0.030	-0.101	0.524	1.000	
-0.043	0.030	-0.119	-0.006	-0.038	0.194	0.361	1.000

Table 24. Eigenvalues computed

Eigenvalues	First differences
4.7350	--
2.1881	2.5469
1.7675	0.4206
1.3077	0.4598
0.09618	0.3459
0.8822	0.0796

Variable X_{16} was introduced to account for the possibility that changes in scores might be an alternative output measure with exploring. Cohn analyzed X_{16} (14). Kiesling did not because of the possibility of "topping out" (43). Variable X_{17} was defined in order to account for the type of student entering the process; 45.7 percent, 64.7 percent and 18.9 percent of the variance of variables X_{15} through X_{17} was explained by the factor resolution. The score level (X_{15}) and the change in score (X_{16}) were related to many of the other variables through F_1. Variables X_{15} and X_{16} are elements of F_1. Teacher salaries (variables X_6 and X_7) are elements in this factor and are positively correlated to it. Other significant positively correlated elements are percent of county general fund devoted to education (X_1) and median family income in the county (X_4), both of which are also positively related to the factor. Increases in variables X_4, X_6, and X_7 are associated with increases in the achievement variables (X_{15}

Table 25. Results of factor analyses (N = 510)

Description of variable	F_1	F_2	F_3	F_4	F_5	h^2	d^2
1. Percent of general fund to public education	0.21	-0.18	0.10	0.09	0.20	0.133	0.867
2. Number of additional high school units required	0.03	0.63	-0.35	0.13	-0.25	0.603	0.397
3. Number of high school units offered	0.14	0.78	-0.37	0.02	-0.15	0.785	0.215
4. Median family income	0.34	0.18	-0.11	-0.10	0.30	0.264	0.736
5. Size	0.14	0.68	-0.59	0.09	-0.20	0.890	0.110
6. Average teacher salary - elementary	0.38	0.73	-0.21	-0.06	0.16	0.744	0.256
7. Average teacher salary - high school	0.34	0.72	-0.10	0.01	0.10	0.652	0.348
8. Per pupil building value	0.07	0.20	0.36	0.00	0.06	0.177	0.823
9. Per pupil spending in general control	-0.14	-0.37	0.48	0.03	0.03	0.385	0.615
10. Per pupil spending in instruction	0.05	0.34	0.74	0.02	0.05	0.675	0.325
11. Per pupil spending on auxiliary services	-0.36	-0.31	0.44	0.05	-0.21	0.458	0.542
12. Per pupil spending in operation of plant	-0.11	0.03	0.75	-0.02	0.02	0.568	0.432
13. Per pupil spending in maintenance	-0.08	0.10	0.29	0.02	-0.07	0.107	0.893
14. Per pupil spending in fixed charges	-0.14	-0.01	0.42	-0.01	-0.06	0.200	0.800
15. ITED score, seniors	0.43	0.12	-0.02	0.51	0.07	0.457	0.543
16. Twelfth ITED - Tenth ITED	0.39	0.01	-0.10	0.69	-0.05	0.647	0.353
17. Change in composite/change in vocabulary	0.17	-0.12	-0.04	0.38	-0.03	0.189	0.811

and X_{16}). Thus, there is some measurable linear connection between expenditures (teacher salaries) and educational output. The number of additional high school units requires (X_2), high school units offered (X_3), size of school (X_5), elementary and high school teacher salaries (X_6 and X_7), building value per pupil (X_8) and instructional expenditures per student (X_{10}) can be grouped together and are positively related to the second factor. Auxiliary and control expenditures per student (X_{11} and X_9) are negatively associated with F_1. In the regression analysis of the survey discussed previously, a significant relation was found between achievement and teacher salaries, but not between achievement and expenditures, per student, of which instructional expenditure is a major part. It can be observed that instructional expenditures and salaries fall in the same factor group, (F_2). Both are positively correlated to the second factor. This indicates that there can be a measurable positive association between achievement, teacher salary, instructional expenditure per student and total expenditure per study.

Two items can be said to be perturbing the relationship. First, there is the negative signs of the auxiliary and control expenditure loadings. This means additional expenditures in these categories are not associated with increased achievement scores, although they increase expenditure per student. Second, the teacher pupil ratio can, as explained by Kiesling (44), have a perverse effect. Increasing the teacher pupil ratio can increase expenditure per student without increasing achievement. A negative relation between the teacher pupil ratio and output can explain why teacher salaries (X_6 and X_7) have a much higher positive loading on F_1, than does instructional expenditures (X_{10}). Thus, increases in achievement arise from hiring better (more expensive) teachers, not from increasing the teacher pupil ratio. Given a budget constraint, it is more effective to hire fewer good teachers than more lesser quality instructors. Decreasing the teacher pupil ratio, decreases the expenditure per pupil, but according to the factor analysis, increases pupil performance. Presumably there is some lower bound to the teacher to ratio given present technology, but its specification would require further study.

The various categories of expenditure per student along with building value per student are loaded in F_3. The combination of positive expenditure loadings, negative loadings on size (X_5), course units (X_2 and X_3) indicate the existence of economies of scale in the educational process. As the scale increases, measured either by enrollment or course offerings, expenditure per student decreases. This finding is consistent with the implied increase in efficiency obtained by decreasing the teacher pupil ratio and hiring better teachers.

The three measures of output (X_{15}, X_{16}, and X_{17}) form the components falling into F_4. The small loading of Variable X_{17} indicates that it is measuring a different aspect of quality of output than are Variables X_{15} and X_{16}. As previously mentioned, a small amount of Variable X_{17}'s variance is explained by the factor rotation. Variable X_{17} is not related to any of the non-achievement variables.

F_5 includes three community setting variables, percent of general fund to public education (X_1), which can be considered an index of the counties' attitude toward education, median county income (X_2) and size of school (X_5). However, the signs of the loadings do not lend themselves to a consistent interpretation.

F_1, the general factor, sometimes considered to be the factor transcending all of the other factors, can be labeled the input-output factor because it contains the output measures, the instructional input measures and the indices of community attitude toward and ability to pay for education. The second and third factors can be considered"input or process factors." F_2 is the resource or real process factor in that it includes course offerings and instructor variables. F_3 can be called the monetary or expenditure process factor in that it includes the financial outlays of the educational process. F_4 is the achievement factor. F_5 is the community setting factor.

The results of factor analysis reports above converge with the findings reported by Ayer (61). In each case the first factor includes the wealth or income variable, expenditure related variables (expenditure per student and teacher salaries) and the output measure (the process quality measures serving as proxies for educational output in Ayer's study and the direct output quality measure in the one previously reported). In both instances there is a population factor or community characteristic, F_2, in Ayer's and F_3 in the one presented above. Economies of scale which appear in the expenditure process factor (F_3) above, did not appear directly in Ayer's analysis. Expenditure per student was located in Ayer's F_1 and size in Ayer's F_3. There is an indirect indication of scale effects in that his process quality measures were also located in the factor containing size variables.

With respect to the specification of educational output-input relations, there are two basic differences in the factor analyses findings. Ayer found a direct relation between expenditures per student and quality of process, which is an indirect measure of output. In the above factor analysis, there is reported an indirect relation (via salaries and the teacher-pupil ratio) between expenditures per student and achievement test score, which is a direct measure of educational output.

Factor analysis is considered to be a source of hypotheses. The hypothesis that the relation between expenditure per student and output is mediated by teacher salaries and the teacher pupil ratio was investigated by using some simple correlations computed by Thomas (77) on identical Iowa data to generate partial correlations. The following variables were used: variable one stood for total expenditure per student, variable two for average daily attendance, variable three for average elementary school salary, variable four average high school salary and variable five for assignments per teacher. The following partial correlations were computed: $r_{1,2} = -0.04$, $r_{1,3.2} = 0.0266$, $r_{1,4.2} = 0.0407$, $r_{1,5.2} = 0.3655$. The coefficients are rather small, but a speculative interpretation may be made on their signs. As size of student body increases, cost per student decreases. When the effect of size is held constant, as elementary salary and high school salary increase, cost per student increase, and assignments per teacher increase. An increase in assignments per teacher is consistent with an increase in the pupil-teacher ratio.

Summary of Specification Problem

The difficulty of specifying educational input-output functions was discussed in this chapter. Three diagnostic hypotheses as to the causes of problem were formulated and discussed. Heteroscedasticity (Hypothesis one, p. 98) was found to exist for urban areas, but was not present for rural or for a group representing both urban and rural sectors. Its presence for urban data could account for understated regression coefficients. In three instances (rural, urban, where a special test was used, and total), there was a significant difference in achievement test means when classified by expenditure level. When only data on average expenditures per student were available, the use of school average test score yielded higher regression coefficients than when individual test scores were used, (Hypothesis two, p. 102). Factor analysis did help to uncover the existence of relations between output and expenditures (Hypothesis three, p. 106).

Application of factor analysis to narrower breakdowns of input characteristics should be fruitful in work preliminary to the specification of input-output functions. It is recommended that this technique be used in the investigation of the relation between achievement test scores and the microeconomic measures of the quality of the educational process (i.e. Growing Edge or Time Scale). The purpose of this investigation would be to specify the link between output and expenditures through process quality measures. This recommendation is based upon the close relation between expenditures and process quality found by other researchers. If diagnostic hypotheses concerning the link between achievement and the process measures are accepted, it is recommended that individual school data be analyzed by regression analysis to determine the impact of changes in resource allocation within the educational system.

CHAPTER VI. SUMMARY AND CONCLUSIONS OF THE ANALYSIS

In this thesis three aspects of the economics of education were presented. These aspects were (1) the resource redistribution effects of state aid to primary and secondary education, (2) the educational output effects of state aid plans, and (3) the specification of the relationship between educational output and input.

In the discussion of the first aspect, six basic types of state general aid grants to public schools were delimited. They were (1) fixed, equalizing; (2) fixed, nonequalizing; (3) variable, equalizing; (4) variable, nonequalizing; (5) percent, equalizing; and (6) percent nonequalizing. It was observed that there were regional patterns to the array of state aid grants used. The percent equalizing state aid grants predominated in the mideastern states as of the 1962-63 school year. Percent equalizing state grants showed the largest increase in use over the period from 1957 to 1962. The fixed unit, nonequalizing grant was the most prevalent form for allocation of state general aid.

The redistribution effects of state grants were defined as equity (equality of burden and opportunity), property tax relief, efficiency of local expenditures and the stimulation of local spending on education. The empirical analysis focused on the property tax relief and equity effects of state grant plans. With respect to these redistribution effects two hypotheses were considered: (1) that the alternative forms in which state school aid can be allocated have differential resource redistribution effects dependent on the economic-demographic environment, and (2) that the redistribution effects of the alternative allocation methods are affected by the form of finance employed by the state for funding school aid.

Four plans were used to test the above hypotheses on recent data from six Iowa counties. The counties were classified as urban, mixed and rural. Plan 1 was representative of the fixed nonequalizing type, while Plans 2 and 3 were representative of the percent equalizing type. Plan 4 was a combination of two types, fixed nonequalizing and variable equalizing.

An amount of state aid to be allocated to the six counties was specified. The state aid to each district in the six counties was calculated for each of the four plans. District millage requirements were determined for each of the four formulas. On an inspection basis there appeared to be differential tax relief impact for each plan. However, there was little uniformity as to the within county (district) property tax effects of plans in counties of similar economic structure. To test the first hypothesis, a two way analysis of variance was performed to determine the statistical significance of the plan and of the geographic unit (county or district) on millage levels. When the geographic effect was accounted for, there was no significant difference in the millage effects of Plans 1, 3, and 4 within counties. Only Plan 2 had a consistently significant effect. With exception of this percentage equalizing plan (Plan 2), none of the plans exhibited a consistent statistically significant differential impact on property tax relief between counties. Within one rural county, Plan 1 produced significantly lower millage levels than Plans 3 or 4. Both over counties and within counties, the county and district effects respectively did have a significant impact on millage. This demonstrated the diversity that existed in the sample counties.

Plan 2 had a within county district equalization feature. This feature did provide a measure of tax relief to large (dominant) districts in urban and rural counties, but not in mixed counties.

Equity (equality of educational opportunity) was defined as equal distribution of educational services and educational burden. In this study, only burden was considered. Local tax askings per student was used as a proxy for burden. The effect of state aid plans on equity was investigated by determining whether the institution of a particular plan equalized the burden within districts. A one way analysis of variance was performed for each plan in which the mean burdens across counties were compared. There were similar (equalized) burdens across counties when Plan 1 and Plan 2 were instituted.

To test the second hypothesis, the district net gain or loss per student was calculated on the assumption that the state aid to the six counties was to be financed in entirety by either the sales tax or the income tax. For the six counties taken together and separately, the mean gain or loss under the income tax was compared with the gain or loss under the sales tax. From this analysis it was concluded that given the form of state aid distribution, the type of finance used to implement the plan was crucial to urban areas. In urban areas, the method of financing state grants made significant differences in loss or gain per student for each of the aid distribution formulas. The tax structure was not as crucial to the plan effect in mixed and rural counties. The greater importance of the form of state aid financing (i.e. the revenue raising side) relative to the form of state aid distribution in urban areas was the most important finding of the first part of the study.

The discussion of the second aspect, the output effects of state aid, focused on the comparison of educational achievement effects of representative plans with achievement outcomes of the optimal distribution of state aid to counties. The optimum distribution of state aid funds was determined by use of a linear programming model. In the model, the objective function to be maximized was the weighted average Iowa Test of Educational Development score achieved by students in the state. The model was constrained to account for local budget limitations and for the requirement that all students must be exposed to some minimum educational experience. The basic sample was expanded to twenty-one Iowa counties.

The output effect of a given state aid plan was evaluated by comparing the manner in which it distributed a specified total amount of aid to the counties with the optimal distribution. The optimal distribution plan was defined as the state aid allocation among counties consistent with the minimum total state aid outlay necessary to achieve specified educational objectives. Quantitative and qualitative methods were used to compare the four representative plans with the optimal plan. The quantitative method involved the use of total deviations and programming deviations. Total deviations were defined as the sum of the per county differences between the amount of money allocated by the optimum and a particular plan. Programming deviations were defined as the sum of per county differences between the amount allocated by the optimum and a particular plan. Only positive differences were considered in the programming deviation frameworks. Those plans associated with the smallest deviations were considered to have superior output effects.

The qualitative method involved the use of nonparametric (rank) correlation measures. Those plans whose fund distribution correlated most highly with the optimal distribution were considered to have superior output effects.

Evaluation of the output effects of plans (based on their relative similarity to the optimum) was dependent on the economic characteristics of the regions considered and on the educational achievement goals specified. For the basic twenty-one county area designed to reflect the distribution and diversity of student population in the State of Iowa, Plan 2, the percentage equalizing plan was superior based on both deviation and programming criteria. Use of the qualitative criteria did not reveal substantial output heterogeneity among plans. Output effects of the plans were similar for urban counties. For rural counties, there was no distinctly superior plans. When minimum score requirements were changed, the output effects also changed. When student migration from rural to urban areas was introduced, no single plan emerged as best from an output point of view.

The analysis of aspects one and two indicated that it is important for educational decision makers of each region to carefully consider both the resource distribution side and the output side of any proposed state grant plan. They must not omit the other distribution effects (stimulation, efficiency and the equitable distribution of educational services) that were not submitted to empirical analysis in this thesis. It is hoped that additional work on these distribution effects of state aid will be forthcoming. Because objectives on the output side and the resource distribution side may be competitive, the decision makers of each region must specify their own priorities. Also, the generation of a set of priorities among the redistribution effects is necessary because they can likewise be competitive.

The notion of an educational production function underlies the analysis of the second aspect. The discussion of the third aspect, the specification of the relation between educational output and input, focused on the difficulties that have been encountered in performing this task. Three hypotheses were explored. Based on the tests of these hypotheses, it was concluded (1) that educational researchers should consider the possibility of heteroscedasticity (nonhomogeneity of variance) in these data as a partial explanation for nonsignificant regression coefficients, (2) that further attempts to disaggregate educational data must be made because of the implications of using average data when data on individuals are appropriate and (3) that factor analysis should be exploited further as a method for uncovering underlying relations between educational output and inputs.

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